Existence of Equilibrium in The Common Agency Model with Adverse Selection

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Outline

1. Motivation
   - Previous Works
   - Contribution

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3. Equilibrium

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   - Contribution
2. Model
3. Equilibrium
4. Results
   - Main Results
   - Basic Ideas for Proofs

Delegation principle.

Martimort (2006):

"What matters per se is not the kind of communication that a principal uses with his agent but the set of options that this principal makes available to the agent."

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- Common agency problem can be analyzed through a menu game

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- Equilibrium must exist!

Related Literature

- Page and Monteiro, JME. (2003):
- Monteiro and Page (2005):

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Contribution

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- Existence of Equilibrium

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- $\Delta(\mathbb{K})$ space of all Borel probability measures over $\mathbb{K}$.
- $\varphi(t, C) \subseteq \Delta(\mathbb{K})$ nonempty compact convex set and $\varphi : T \times P \rightrightarrows \Delta(\mathbb{K})$ continuous correspondence: Set of available choices

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- $I_e \subseteq I$ of principals only allows for exclusive contracts:
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- We can let some $f \in K_i$ denote no contracting

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- We show φ^{PM}, φ^{MP} and φ^{H} are continuous with nonempty, convex and compact values.

Optimal problems
Agent

- Given $t \in T$ and $C \in P$, the agent's problem is

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- A strategy for the agent is then a measurable function $\sigma : T \times P \to \Delta(\mathbb{K})$.
- σ is an optimal strategy if and only if it is a selection of Λ.

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- $\pi_i : T \times \mathbb{K} \to \mathbb{R}$ bounded Carathéodory function: Profit.
- If the principals offer a menu $C = (C_1, \ldots, C_m) \in P$ and the agent uses $\sigma : T \times P \to \Delta(\mathbb{K})$, then payoff is

$$F_i(t, C; \sigma) = \int_{\mathbb{K}} \pi_i(t, k) \mathrm{d}\sigma(k|t, C).$$

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- if principals choose strategies $\alpha = (\alpha_1, \ldots, \alpha_m)$,

$$F_i(\alpha; \sigma) = \int_P \int_T F_i(t, C; \sigma) \mathrm{d}\mu(t) \mathrm{d}\alpha(C) \tag{1}$$

denotes principal i's payoff.

Menu Games

A menu game is denoted by G and we use G^{PM}, G^{MS} and G^{H} to denote particular menu games for the corresponding choices of $\mathbb{K}$ and φ mentioned above. We say that a menu game G is *continuous* if it satisfies all the above assumptions.

Sequential Equilibrium

Definition

An assessment $(\nu, (\alpha, \sigma))$ is a sequential equilibrium of a menu game G if and only if

1. $\nu_i = \mu \times \alpha_1 \times \cdots \times \alpha_{i-1}$ for all $i \in I$,
2. σ is a measurable selection of Λ and
3. $F_i(\alpha; \sigma) \geq F_i(\bar{\alpha}_i, \alpha_{-i}; \sigma)$ for all $i \in I$ and $\bar{\alpha}_i \in \Delta(P_i)$.

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Thus, in a sequential equilibrium of G, beliefs are determined by Bayes' rule, the agent optimizes for all possible types and menus offered, and each principal optimizes given the strategy of the other principals and the strategy of the agent.

Page and Monteiro (2003)

Principal i's payoff function $F_i^{PM} : P \to \mathbb{R}$ is the expected value of π^*:

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Principals' beliefs are irrational: in general, there is no agent's strategy that justifies those beliefs.

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Let for all $t \in T$ and $C \in P$

$$H(t, C) = \{i \in I : \text{there exists } f \in C_i \text{ such that } \delta_{(i,f)} \in \Lambda(t, C)\},$$

be the set of principals that have offered a contract in her optimal choice correspondence.

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Principal i's payoff function:

$$F_i^{MP}(C) = \int_T \frac{\pi_i^*(t, C)}{|H(t, C)|} \mathrm{d}\mu(t), \forall i \in I \text{ and } C \in P. \tag{3}$$

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Implicitly, in a Monteiro-Page equilibrium, the agent's strategy is fixed exogenously. It assumes that the agent uses strategy σ^{MP}

$$\sigma^{MP}\left(\{i\}\times\{f \in C_i : \delta_{(i,f)} \in \Lambda(t, C) \text{ and } \pi_i(t,i,f) = \pi^*(t,C)\}\middle|t, C\right) = \frac{1}{|H(t,C)|}$$

Fajardo and Carmona (2006)

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  Without the no-fixed cost property: $\pi_i(t, j, f) = 0$ for all $i, j \in I$, $i \neq j$, $t \in T$ and $f \in C_i$.

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Fajardo and Carmona (2006)

Remark

Let G be a menu game with the no-fixed-cost property. Then,

- $F_i^{MP}(\alpha) = F_i(\alpha; \sigma^{MP})$, *for all* $i \in I$, $\alpha \in \Delta(P)$ *and all Monteiro-Page strategies* σ^{MP}.
- *If* α *is a Monteiro-Page equilibrium, then* (α, σ^{MP}) *is a sequential equilibrium strategy for every Monteiro-Page strategy* σ^{MP}.
- *If* $\Lambda(t, C)$ *is singleton for all* $t \in T$ *and* $C \in P$ *and* (α, σ) *is a sequential equilibrium strategy, then* α *is a Monteiro-Page equilibrium (equal to Page-Monteiro equilibrium) and* σ *is a Monteiro-Page strategy.*

Existence of Equilibrium

Theorem

A sequential equilibrium exists for all continuous games G.

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Corollary

All continuous games G^H, G^{PM} and G^{MS} have a sequential equilibrium.

Generalization of Simon and Zame (1990)

Theorem

A solution exists for all generalized games with an endogenous sharing rule.

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- We dispense the exclusivity and the no-fixed-cost assumptions.
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- Informed principals
- Other models with endogenous sharing rules.

References I

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