Duality and Derivative Pricing with Time-Changed Lévy Processes

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Fourth Bachelier Finance Society Congress. Tokyo, August 20, 2006

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- Bidimensional Derivative pricing

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- Conclusions

Motivation

BiDimensional Derivative Pricing:

	European type	American Perpetual type
BM	Margrabe (1978)	Gerber and Shiu (1996)
LP	Fajardo and Mordecki (2006)	Fajardo and Mordecki (2006)
AP	Eberlein and Papantaleon (2005b)	?
TCLP	This Paper	?

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Duality

	Put-Call	Exotic Derivatives
BM	Carr and Chesney (1996)	Henderson and Wojakowski (2002)
LP	Fajardo and Mordecki (2006)	Eberlein and Papantaleon (2005a)
AP	Eberlein and Papantaleon (2005b)	?
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SM	Schroder (1999)	?

Lévy Processes

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- $X_0 = 0$, and has independent increments, given $0 < t_1 < t_2 < ... < t_n$, the r.v.

$$X_{t_1}, X_{t_2} - X_{t_1}, \cdots, X_{t_n} - X_{t_{n-1}}$$

are independents.

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  $$X_{t_1}, X_{t_2} - X_{t_1}, \cdots, X_{t_n} - X_{t_{n-1}}$$

  are independents.
- The distribution of the increment $X_t - X_s$ is homogenous in time, that is, depends just on the difference $t - s$.

Lévy-Khintchine Formula

A key result in the theory of Lévy Processes is the Lévy-Khintchine formula, that computes de characteristic function of X_t as:

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Where ψ is called *characteristic exponent*, and is given by:

$$\psi(z) = bz + \frac{1}{2}\sigma^2 z^2 + \int_{I\!R} (e^{zy} - 1 - zy\mathbf{1}_{\{|\mathbf{y}|<\mathbf{1}\}})\mathbf{\Pi}(\mathbf{dy}),$$

where b and $\sigma \geq 0$ are real constants, and Π is a positive measure in $I\!R - \{0\}$ such that $\int (1 \wedge y^2)\Pi(dy) < \infty$, and is called Lévy measure,

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- Due to the homogeneity in time we can not have flexible representations for the term structure of volatility.

This Paper

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- Derivative Pricing and Duality Relationship.

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- Correlation between Lévy processes and random clock$\rightarrow$ leverage effect.
- Original clock as calendar time and new random clock as business time

Time-Change

Now let $t \mapsto \mathcal{T}_t, \quad , t \geq 0$, be an increasing RCLL process, such that for each fixed t, $\mathcal{T}_t$ is a stopping time with respect to $\mathbf{F}$.

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Furthermore, suppose $\mathcal{T}_t$ is finite $P - a.s., \ \forall t \geq 0$ and $\mathcal{T}_t \to \infty$ as $t \to \infty$.

Then $\{\mathcal{T}_t\}$ defines a random change on time, we can also impose $E\mathcal{T}_t = t$.

Time-Changed Lévy processes

Then, consider the process Y_t defined by:

$$Y_t \equiv X_{\mathcal{T}_t}, \quad t \geq 0,$$

using different triplets for X and different time changes $\mathcal{T}_t$, we can obtain a good candidate for the underlying asset return process.

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A more general situation is when $\mathcal{T}_t$ is modeled by a non-decreasing semimartingale.

Time-Changed Lévy Processes

In that case

$$\mathcal{T}_t = b_t + \int_0^t \int_0^\infty y\mu(dy, ds)$$

where b is a drift and μ is the counting measure of jumps of the time change and we can take $\mu = 0$ and just take locally deterministic time changes, so we need to specify the local intensity ν:

$$\mathcal{T}_t = \int_0^t \nu(s_-)ds \qquad (1)$$

where ν is the instantaneous activity rate, observe that ν must be non-negative.

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- When X_t is a pure jump Lévy process, ν is proportional to the Lévy intensity of jumps.

Time-Changed Lévy Processes

We can obtain the characteristic function of Y_t:

$$\phi_{Y_t}(z) = \mathsf{E}(e^{z'X_{\mathcal{T}_t}}) = \mathsf{E}\left(\mathsf{E}\left(e^{z'X_u}/\mathcal{T}_t = u\right)\right).$$

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If $\mathcal{T}_t$ and X_t were independent, then:

$$\phi_{Y_t}(z) = \mathcal{L}_{\mathcal{T}_t}(\psi(z))$$

where $\mathcal{L}_{\mathcal{T}_t}$ is the Laplace transform of $\mathcal{T}_t$.

Time-Changed Lévy Processes

So if the Laplace transform of $\mathcal{T}$ and the characteristic exponent of X have closed forms, we can obtain a closed form for ϕ_{Y_t}. Using equation (1) we have:

$$\mathcal{L}_{\mathcal{T}_t}(\lambda) = \mathsf{E}(e^{-\lambda \int_0^t \nu(s_-)ds}) \tag{3}$$

From bond pricing literature Carr and Wu (2004) obtained a closed forms for ϕ_{Y_t}.

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- When $\mathcal{T}$ has independent increments we can obtain a deterministic characteristics for Y.
- When $\mathcal{T}$ is just a processes that turns Y in to a general semimartingale we can obtain a predictable version of the characteristics.

Model

Consider a market model with three assets (S^1, S^2, S^3) given by

$$S_t^1 = e^{Y_t^1}, \quad S_t^2 = S_0^2 e^{Y_t^2}, \quad S_t^3 = S_0^3 e^{Y_t^3} \tag{4}$$

where (Y^1, Y^2, Y^3) is a three dimensional Time-Changed Lévy process.

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where (Y^1, Y^2, Y^3) is a three dimensional Time-Changed Lévy process.
Consider a function:

$$f : (0, \infty) \times (0, \infty) \to I\!R$$

homogenous of an arbitrary degree α; We introduce a derivative contract with payoff given by

$$\Phi_t = f(S_t^2, S_t^3)$$

Problem

We want to price that derivative contract. In the European case, the problem reduces to the computation of

$$E_T = E(S_0^2, S_0^3, T) = \mathsf{E}\left[e^{-Y_T^1} f(S_0^2 e^{Y_T^2}, S_0^3 e^{Y_T^3})\right] \qquad (6)$$

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In the American case, if $\mathcal{M}_T$ denotes the class of stopping times up to time T, i.e:

$$\mathcal{M}_T = \{\tau :\ 0 \leq \tau \leq T,\ \tau \text{ stopping time}\}$$

for the finite horizon case, putting $T = \infty$ for the perpetual case. Hence we need to find the value function A_T and an optimal stopping time τ^* in $\mathcal{M}_T$.

Dual market Method

such that

$$A_T = A(S_0^2, S_0^3, T) = \sup_{\tau \in \mathcal{M}_T} \mathsf{E}\left[e^{-Y_\tau^1} f(S_0^2 e^{Y_\tau^2}, S_0^3 e^{Y_{\tau^3}^3}) \right]$$

$$= \mathsf{E}\left[e^{-Y_{\tau^*}^1} f(S_0^2 e^{Y_{\tau^*}^2}, S_0^3 e^{Y_{\tau^*}^3}) \right].$$

Dual market Method

Observe that

$$e^{-Y_t^1} f(S_0^2 e^{Y_t^2}, S_0^3 e^{Y_t^3}) = e^{-Y_t^1 + \alpha Y_t^3} f(S_0^2 e^{Y_t^2 - Y_t^3}, S_0^3).$$

Let $\rho = -\log \mathsf{E} e^{-Y_1^1 + \alpha Y_1^3}$, that we assume finite. The process

$$Z_t = e^{-Y_t^1 + \alpha Y_t^3 + \rho \mathcal{T}_t} \qquad (8)$$

is a density process (i.e. a positive martingale starting at $Z_0 = 1$) that allow us to introduce a new measure $\tilde{P}$ by its restrictions to each $\mathcal{F}_t$ by the formula

$$\frac{d\tilde{P}_t}{dP_t} = Z_t.$$

Dual market Method

Denote now by $Y_t = Y_t^2 - Y_t^3$, and $S_t = S_0^2 e^{Y_t}$. Finally, let

$$F(x) = f(x, S_0^3).$$

With the introduced notations, under the change of measure we obtain

$$E_T = \tilde{\mathsf{E}}\left[e^{-\rho \mathcal{T}_T} F(S_T)\right]$$

$$A_T = \sup_{\tau \in \mathcal{M}_T} \tilde{\mathsf{E}}\left[e^{-\rho \mathcal{T}_\tau} F(S_\tau)\right]$$

Example

Let S_T^2 and S_T^3 be two risky assets, a contract with payoff $\Phi_t = (S_T^2 - S_T^3)^+$ can be priced using *The Dual Market Method*:

$$D = \mathsf{E}\left[e^{-rT}(S_T^2 - S_T^3)^+\right].$$

$$= \int_{\mathcal{A}} e^{-rT}(S_0^2 e^{Y_T^2} - S_0^3 e^{Y_T^3})dP$$

Assuming for simplicity $S_0^2 = S_0^3 = 1$,
Then $\mathcal{A} = \{\omega \in \Omega : Y_T^2(\omega) > Y_T^3(\omega)\}$, we apply the method:

Example

$$D = \int_{\mathcal{A}} e^{-rT}(e^{Y_T^2} - e^{Y_T^3})dP = \int_{\{S_T>1\}} e^{-rT} e^{Y_T^3}(S_T - 1)dP$$

where $S_T = e^{Y_T}$ and $Y = Y^2 - Y^3$.

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where $S_T = e^{Y_T}$ and $Y = Y^2 - Y^3$. Now the dual measure: $\rho = -\log \mathsf{E}e^{-r+Y_1^3} = r - \log \mathsf{E}e^{Y_1^3}$, then:

$$d\widetilde{P} = \frac{e^{r(t-\mathcal{T}_t)+Y_T^3}}{(\mathsf{E}e^{Y_1^3})^{\mathcal{T}_T}}dP.$$

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To simplify assume $r = 0$, then:

$$D = \int_{\{S_T>1\}} e^{-\rho\mathcal{T}_T} S_T d\widetilde{P} - \int_{\{S_T>1\}} e^{-\rho\mathcal{T}_T} d\widetilde{P}$$

Duality

Consider a market with two assets given by

$$S_t^1 = e^{Y_t}, \text{ and } S_t^2 = S_0^2 e^{rt}$$

where (Y) is a one dimensional Time-Changed Lévy process, and for simplicity, and without loss of generality we take $S_0^1 = 1$.

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$$B_t = (r - \delta)t - \frac{1}{2}\int_0^t c_s ds - \int_0^t \int_{\mathbb{R}} (e^x - 1 - x)\nu(ds, dx),$$

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where,

$$B_t = \int_0^t b_s ds \text{ and } C_t = \int_0^t c_s ds,$$

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Duality

Let us assume that τ is a stopping time with respect to the given filtration $\mathcal{F}$, that is $\tau\colon \Omega \to [0,\infty]$ belongs to $\mathcal{F}_t$ for all $t \geq 0$; and introduce the notation

$$\mathcal{C}(S_0, K, r, \delta, \tau, \Psi) = \mathsf{E}e^{-r\tau}(S_\tau - K)^+ \tag{9}$$

$$\mathcal{P}(S_0, K, r, \delta, \tau, \Psi) = \mathsf{E}e^{-r\tau}(K - S_\tau)^+ \tag{10}$$

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$$\mathcal{P}(S_0, K, r, \delta, \tau, \Psi) = \mathsf{E} e^{-r\tau}(K - S_\tau)^+ \tag{12}$$

If $\tau = T$, where T is a fixed constant time, then formulas (9) and (10) give the price of the European call and put options respectively.

Put-Call Duality

Proposition 0.1 *Consider a Time-changed Lévy market with driving process Y with characteristic triplet $\Psi = (B, C, \nu)$. Then, for the expectations introduced in* (9) *and* (10) *we have*

$$\mathcal{C}(S_0, K, r, \delta, \tau, \Psi) = \mathcal{P}(K, S_0, \delta, r, \tau, \tilde{\Psi}), \quad (13)$$

where $\tilde{\Psi}(z) = (\tilde{B}, \tilde{C}, \tilde{\nu})$ is the characteristic triplet (of a certain semimartingale) that satisfies:

$$\begin{cases} \tilde{B}_t & = (\delta - r)t - \frac{1}{2}\int_0^t c_s ds - \int_0^t \int \left(e^x - 1 - x\mathbf{1}_{\{|x|\leq 1\}}\right)\tilde{\nu}(ds, dx), \\ \tilde{C} & = C, \\ \tilde{\nu}(dy) & = e^{-y}\nu(-dy) \end{cases}$$

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Since P is the risk neutral probability, the process B is also determined by the other characteristics.

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where (B, C, ν) is the triplet characteristics of Y and h is a truncation function.

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Gyrsanov's Theorem for semimartingales:

$$\tilde{B}_t = -B_t - C_t - \int\int h(x)(e^x - 1)\nu(ds, dx) \qquad (17)$$

Symmetric markets

Lets define symmetric markets by

$$\mathcal{L}\Big(e^{-(r-\delta)t+Y_t} \mid P\Big) = \mathcal{L}\Big(e^{-(\delta-r)t-Y_t} \mid \tilde{P}\Big), \tag{18}$$

In view of (0.1), we have

$$\nu(dy) = e^{-y}\nu(-dy). \tag{19}$$

This ensures $\tilde{\nu} = \nu$, and from this follows

$$B - (r-\delta) = \tilde{B} - (\delta - r)$$

, giving (18), as we always have $\tilde{C} = C$.

Bates's $x\%$-Rule

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If $r = \delta$, we can take the future price F as the underlying asset in Proposition 1.

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If $r = \delta$, we can take the future price F as the underlying asset in Proposition 1.

Corollary 0.3 *Take $r = \delta$ and assume (19) holds, we have*

$$\mathcal{C}(F_0, K_c, r, \tau, \Psi) = (1 + x)\, \mathcal{P}(F_0, K_p, r, \tau, \Psi), \qquad (22)$$

where $K_c = (1 + x)F_0$ and $K_p = F_0/(1 + x)$, with $x > 0$.

Conclusions

Time-Changed Lévy Processes:

- Bidimensional derivatives

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Extensions:

- Duality with Exotic Derivatives
- Skewness Premium
- Perpetual American Option with TCLP

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Triplet for Semimartingales

When Y_t has indep. increments. and it's distribution is inf. divisible: $E(e^{iuY_t}) = e^{\psi_t(u)}$, with

$$\psi_t(u) = iub_t - \frac{u^2}{2}c_t + \int (e^{iux} - 1 - iuh(x))F_t(dx)$$

$b_t \in I\!R,\ c_t \in I\!R_+$ and F_t a positive measure which integrates $x^2 \wedge 1$. h is abounded Borel function with compact support which behaves like "x" near the origin, the independence of increments imply:

$$\frac{e^{iuY_t}}{e^{\psi_t(u)}}, \quad \text{is a martingale}$$

If Y is a semimartingale, we can find a unique (B, C, ν) such that if we use this triplet instead if (b, c, F) to define ψ_t, we still have the martingale property.

Gyrsanov for Semimartingales

Lemma 0.1 *Let Y be a d-dimensional semimartingale with finite variation with triplet (B, C, ν) under P, let u, v be vectors in ^{d} and $v \in [-M, M]^d$. Moreover let $\tilde{P} \sim P$, with density $\frac{d\tilde{P}}{dP} = \frac{e^{<v, Y_T>}}{\mathsf{E}[e^{<v, Y_T>}]}$. Then the process $Y^* :=< u, Y >$ is a $\tilde{P}-$semimartingale with triplet:*

$$
\begin{aligned}
b_s^* &= < u, b_s > + \frac{1}{2}(< u, c_s v > + < v, c_s u >) + \int_d < u, x > (e^{<v,x>} - 1)\lambda_s(dx) \\
c_s^* &= < u, c_s u > \\
\lambda_s^* &= \Lambda(\kappa_s)
\end{aligned}
$$

where Λ is a mapping $\lambda : I\!R^d \to I\!R$ such that $x \mapsto \Lambda(x) =< u, x >$ and κ_s is a measure defined by $\kappa_s(A) = \int_A e^{<v,x>} \lambda_s(dx)$.

Stationarity and Independence

Lemma 0.2 *Let* $\{X_t\}$ *be a Lévy process and* $\{\tau_t\}_{t\leq T}$ *be an independent increasing cádlág process with stationary increments. Then* $\{X_{\tau_t}\}$ *has stationary increments.*

Stationarity and Independence

Lemma 0.3 *Let $\{X_t\}$ be a Lévy process and $\{\tau_t\}_{t\leq T}$ be an independent increasing cádlág process with stationary increments. Then $\{X_{\tau_t}\}$ has stationary increments.*

Lemma 0.4 *Let $\{X_t\}$ be a Lévy process such that , for any $t \geq 0,\ EX_t^2 < \infty$ and $EX_t = 0$. Let $\{\tau_t\}_{t\leq T}$ be and independent cádlág process such that , for any $t \geq 0,\ E\tau_t < \infty$. Then, for any $t \geq 0,\ EX_{\tau_t}^2 < \infty$ and $EX_{\tau_t} = 0$. Moreover, the increments of X_{τ_t} over disjoint intervals are not correlated.*

See Cherny and Shiryaev (2002). Ch. 5, Lemma 5.2 and 5.3.