

Skewness Premium with Lévy Processes

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Outline

- 1 Option Market
- 2 Lévy Processes
- 3 Duality
- 4 Symmetry
- 5 Skewness Premium

- Observed moneyness biases in American call and put options: S&P500 options traded on CMEX

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- American Foreign currency call options traded in Philadelphia Stock Exchange

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- American Foreign currency call options traded in Philadelphia Stock Exchange
- The Biases are not in the same direction, nor are they constant over time.

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- Call and Put prices directly reflects characteristics of the upper and lower tails of the risk neutral distribution.
- Then relative prices of OTM options will reflect the skewness of the risk neutral distribution.

Put-Call Parity:

$$p + S = c + Xe^{-rT}$$

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Just for European Options!

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Put-Call Duality:

$$c(S_0, K, r, \delta, \tau, \psi) = p(K, S_0, \delta, r, \tau, \tilde{\psi})$$

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European and American Options! **Different Strike**

From Duality

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Where F is the future price;

$$K_c = F(1 + x) \text{ and } K_p = F/(1 + x). \text{ i.e. } K_c K_p = F^2.$$

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Bates' $x\%$ rule!

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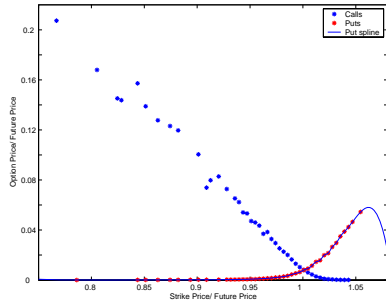
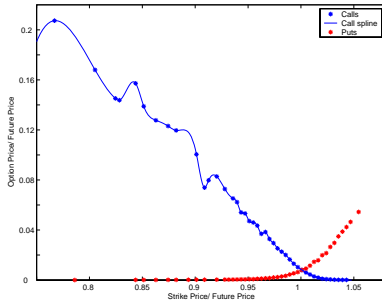
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- For which parameters $SK = \frac{C}{P} - 1 \leq 0$?

Option Market Data: 08/31/2006



Option Prices on S&P500, T=09/15/2006.

OTM options S&P500 - 08/31/06. T=09/15/06, F=1303.82

K_C	$K_p = F^2/K_C$	$x = K_C/F - 1$	$x_{obs} = C_{obs}/p_{int} - 1$	$X - X_{obs}$
1305	1302.641	0.00091	0.61456	-0.61366
1310	1297.669	0.00474	0.53280	-0.52806
1315	1292.735	0.00856	0.42730	-0.41872
1320	1287.838	0.01241	0.10891	-0.09650
1325	1282.979	0.01625	-0.11658	0.13283
1330	1278.155	0.02008	-0.45097	0.47105
1335	1273.368	0.02392	-0.50378	0.52770
1340	1268.617	0.02775	-0.61306	0.64081
1345	1263.901	0.03158	-0.73872	0.77031
1350	1259.220	0.03542	-0.81448	0.84990
1355	1254.573	0.03925	-0.80297	0.84222
1360	1249.961	0.04309	-0.82437	0.86745

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K_p	$K_c = F^2/K_p$	$x = F/K_p - 1$	$x_{obs} = c_{int}/p_{obs} - 1$	$x - x_{obs}$
1250	1359.957	0.043056	-0.88837	0.931421
1255	1354.539	0.0389	-0.86897	0.907873
1260	1349.164	0.034778	-0.85655	0.891331
1265	1343.831	0.030688	-0.78107	0.81176
1270	1338.541	0.02663	-0.70531	0.731941
1275	1333.291	0.022604	-0.63926	0.661869
1280	1328.083	0.018609	-0.51726	0.535865
1285	1322.916	0.014646	-0.31216	0.326801
1290	1317.788	0.010713	-0.20329	0.214005
1295	1312.7	0.006811	-0.03659	0.043397
1300	1307.651	0.002938	0.090739	-0.0878

ITM options S&P500 - 08/31/06. T=09/15/06, F=1303.82

K_c	$K_p = F^2/K_c$	$x = K_c/F - 1$	$x_{obs} = c_{obs}/p_{int} - 1$	$x - x_{obs}$
1230	1382.070	-0.05662	0.05068	-0.10730
1235	1376.475	-0.05278	0.13642	-0.18920
1240	1370.925	-0.04895	0.11501	-0.16395
1245	1365.419	-0.04511	0.19770	-0.24281
1250	1359.957	-0.04128	0.27794	-0.31922
1255	1354.539	-0.03744	0.28073	-0.31817
1260	1349.164	-0.03361	0.53629	-0.56990
1265	1343.831	-0.02977	0.57498	-0.60476
1270	1338.541	-0.02594	0.60672	-0.63266
1275	1333.291	-0.02210	0.67537	-0.69748
1280	1328.083	-0.01827	0.69133	-0.70959
1285	1322.916	-0.01443	0.96631	-0.98074
1290	1317.788	-0.01060	0.90484	-0.91544
1295	1312.700	-0.00676	0.79406	-0.80082
1300	1307.651	-0.00293	0.78018	-0.78311

ITM options S&P500 - 08/31/06. T=09/15/06, F=1303.82

K_p	$K_c = F^2/K_p$	$x = F/K_p - 1$	$x_{obs} = c_{int}/p_{obs} - 1$	$x - x_{obs}$
1305	1302.641	-0.0009	0.13084	-0.13175
1310	1297.669	-0.00472	0.25254	-0.25726
1315	1292.735	-0.0085	0.26191	-0.27041
1320	1287.838	-0.01226	0.24282	-0.25507
1325	1282.979	-0.01598	0.34642	-0.36240
1330	1278.155	-0.01968	0.18321	-0.20289
1335	1273.368	-0.02336	0.23799	-0.26135
1340	1268.617	-0.02700	0.14586	-0.17286
1345	1263.901	-0.03062	0.15264	-0.18325
1350	1259.220	-0.03421	0.10121	-0.13542
1355	1254.573	-0.03777	-0.03964	0.00187
1360	1249.961	-0.04131	0.02834	-0.06965
1365	1245.382	-0.04482	-0.01010	-0.03472
1375	1236.325	-0.05177	-0.04510	-0.00667

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- OTM options: Usually, $x_{obs} < x$. That means $\frac{c}{p} - 1 < x$.
- ITM options: Usually, $x_{obs} > x$. That means $\frac{c}{p} - 1 > x$.
- Asymmetry in the market.

Contribution

- Theoretical proposition that quantify the relation between OTM Calls and Puts when the underlying follows a Geometric Lévy Process.

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- Simply diagnostic for judging which distributions are consistent with observed option prices.

Lévy Processes

Consider a stochastic process $X = \{X_t\}_{t \geq 0}$, defined on $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t \geq 0}, \mathbb{Q})$. We say that $X = \{X_t\}_{t \geq 0}$ is a Lévy Process if:

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- X has paths RCLL
- $X_0 = 0$, and has independent increments, given $0 < t_1 < t_2 < \dots < t_n$, the r.v.

$$X_{t_1}, X_{t_2} - X_{t_1}, \dots, X_{t_n} - X_{t_{n-1}}$$

are independents.

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are independents.

- The distribution of the increment $X_t - X_s$ is homogenous in time, that is, depends just on the difference $t - s$.

Lévy-Khintchine Formula

A key result in the theory of Lévy Processes is the Lévy-Khintchine formula:

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$$\psi(z) = az + \frac{1}{2}\sigma^2 z^2 + \int_{\mathbb{R}} (e^{zy} - 1 - zy\mathbf{1}_{\{|y|<1\}}) \Pi(dy),$$

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where a and $\sigma \geq 0$ are real constants, and Π is a positive measure in $\mathbb{R} - \{0\}$ such that $\int (1 \wedge y^2) \Pi(dy) < \infty$, called the *Lévy measure*.

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$$\psi(z) = az + \frac{1}{2}\sigma^2 z^2 + \int_R (e^{zy} - 1 - zy\mathbf{1}_{\{|y|<1\}}) \Pi(dy),$$

where a and $\sigma \geq 0$ are real constants, and Π is a positive measure in $R - \{0\}$ such that $\int (1 \wedge y^2) \Pi(dy) < \infty$, called the *Lévy measure*. The triplet (a, σ^2, Π) is the *characteristic triplet*.

Model

Consider a market with two assets given by

$$S_t^1 = e^{X_t}, \text{ and } S_t^2 = S_0^2 e^{rt}$$

where (X) is a one dimensional Lévy process, and for simplicity, and without loss of generality we take $S_0^1 = 1$.

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In this model we assume that the stock pays dividends with constant rate $\delta \geq 0$, and that the given probability measure \mathbb{Q} is the chosen equivalent martingale measure.

Duality

Denote by \mathcal{M}_T the class of stopping times up to a fixed constant time T , i.e:

$$\mathcal{M}_T = \{\tau : 0 \leq \tau \leq T, \tau \text{ stopping time w.r.t } \mathbf{F}\}$$

for the finite horizon case and for the perpetual case we take $T = \infty$ and denote by $\overline{\mathcal{M}}$ the resulting stopping times set.

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for the finite horizon case and for the perpetual case we take $T = \infty$ and denote by $\overline{\mathcal{M}}$ the resulting stopping times set. Then, for each stopping time $\tau \in \mathcal{M}_T$ we introduce

$$c(S_0, K, r, \delta, \tau, \psi) = \mathbf{E} e^{-r\tau} (S_\tau - K)^+, \quad (1)$$

$$p(S_0, K, r, \delta, \tau, \psi) = \mathbf{E} e^{-r\tau} (K - S_\tau)^+. \quad (2)$$

Duality

For the American finite case, prices and optimal stopping rules τ_c^* and τ_p^* are defined, respectively, by:

$$\begin{aligned} C(S_0, K, r, \delta, T, \psi) &= \sup_{\tau \in \mathcal{M}_T} \mathbf{E} e^{-r\tau} (S_\tau - K)^+ \\ &= \mathbf{E} e^{-r\tau_c^*} (S_{\tau_c^*} - K)^+ \end{aligned} \quad (3)$$

$$\begin{aligned} P(S_0, K, r, \delta, T, \psi) &= \sup_{\tau \in \mathcal{M}_T} \mathbf{E} e^{-r\tau} (K - S_\tau)^+ \\ &= \mathbf{E} e^{-r\tau_p^*} (K - S_{\tau_p^*})^+, \end{aligned} \quad (4)$$

Duality

And for the American perpetual case, prices and optimal stopping rules are determined by

$$\begin{aligned}\bar{C}(S_0, K, r, \delta, \psi) &= \sup_{\tau \in \bar{\mathcal{M}}} \mathbf{E} e^{-r\tau} (S_\tau - K)^+ \mathbf{1}_{\{\tau < \infty\}} \\ &= \mathbf{E} e^{-r\tau_c^*} (S_{\tau_c^*} - K)^+ \mathbf{1}_{\{\tau < \infty\}},\end{aligned}\quad (5)$$

$$\begin{aligned}\bar{P}(S_0, K, r, \delta, \psi) &= \sup_{\tau \in \bar{\mathcal{M}}} \mathbf{E} e^{-r\tau} (K - S_\tau)^+ \mathbf{1}_{\{\tau < \infty\}} \\ &= \mathbf{E} e^{-r\tau_p^*} (K - S_{\tau_p^*})^+ \mathbf{1}_{\{\tau < \infty\}}.\end{aligned}\quad (6)$$

Lemma (Duality)

Consider a Lévy market with driving process X with characteristic exponent $\psi(z)$. Then, for the expectations introduced in (1) and (2) we have

$$c(S_0, K, r, \delta, \tau, \psi) = p(K, S_0, \delta, r, \tau, \tilde{\psi}), \quad (7)$$

where

$$\tilde{\psi}(z) = \tilde{a}z + \frac{1}{2}\tilde{\sigma}^2z^2 + \int_{\mathbb{R}} (e^{zy} - 1 - zh(y))\tilde{\Pi}(dy) \quad (8)$$

is the characteristic **exponent** (of a certain Lévy process) that satisfies

$$\begin{cases} \tilde{a} &= \delta - r - \sigma^2/2 - \int_{\mathbb{R}} (e^y - 1 - h(y))\tilde{\Pi}(dy), \\ \tilde{\sigma} &= \sigma, \\ \tilde{\Pi}(dy) &= e^{-y}\Pi(-dy). \end{cases} \quad (9)$$

Duality

Corollary (European Options)

For the expectations introduced in (1) and (2) we have

$$c(S_0, K, r, \delta, T, \psi) = p(K, S_0, \delta, r, T, \tilde{\psi}), \quad (10)$$

with ψ and $\tilde{\psi}$ as in the Duality Lemma.

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For the expectations introduced in (1) and (2) we have

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with ψ and $\tilde{\psi}$ as in the Duality Lemma.

Corollary (American Options)

For the value functions in (3) and (4) we have

$$C(S_0, K, r, \delta, T, \psi) = P(K, S_0, \delta, r, T, \tilde{\psi}), \quad (11)$$

with ψ and $\tilde{\psi}$ as in the Duality Lemma.

Duality

Corollary (Perpetual Options)

For prices of Perpetual Call and Put options in (5) and (6) the optimal stopping rules have, respectively, the form

$$\begin{aligned}\tau_c^* &= \inf\{t \geq 0: S_t \geq S_c^*\}, \\ \tau_p^* &= \inf\{t \geq 0: S_t \leq S_p^*\}.\end{aligned}$$

where the constants S_c^ and S_p^* are the critical prices. Then, we have*

$$\bar{C}(S_0, K, r, \delta, \psi) = \bar{P}(K, S_0, \delta, r, \tilde{\psi}), \quad (12)$$

with ψ and $\tilde{\psi}$ as in the Duality Lemma. Furthermore, when $\delta > 0$, for the optimal stopping levels, we obtain the relation $S_c^ S_p^* = S_0 K$.*

Dual markets

Given a Lévy market with driving process characterized by ψ , consider a market model with two assets, a deterministic savings account $\tilde{B} = \{\tilde{B}_t\}_{t \geq 0}$, given by

$$\tilde{B}_t = e^{\delta t}, \quad \delta \geq 0,$$

and a stock $\tilde{S} = \{\tilde{S}_t\}_{t \geq 0}$, modelled by

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where $\tilde{X}_t = -X_t$ is a Lévy process with characteristic exponent under $\tilde{\mathbb{Q}}$ given by $\tilde{\psi}$ in (8).

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where $\tilde{X}_t = -X_t$ is a Lévy process with characteristic exponent under $\tilde{\mathbb{Q}}$ given by $\tilde{\psi}$ in (8). The process \tilde{S}_t represents the price of KS_0 dollars measured in units of stock S .

Symmetric markets

Lets define symmetric markets by

$$\mathcal{L}(e^{-(r-\delta)t+X_t} \mid \mathbb{Q}) = \mathcal{L}(e^{-(\delta-r)t-X_t} \mid \tilde{\mathbb{Q}}), \quad (13)$$

meaning equality in law.

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A necessary and sufficient condition for (13) to hold is

$$\Pi(dy) = e^{-y}\Pi(-dy), \quad (14)$$

This ensures $\tilde{\Pi} = \Pi$, and from this follows

$$a - (r - \delta) = \tilde{a} - (\delta - r)$$

, giving (13), as always $\tilde{\sigma} = \sigma$.

Bates' $x\%$ -Rule

If the call and put options have strike prices $x\%$ out-of-the money relative to the forward price, then the call should be priced $x\%$ higher than the put.

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Corollary

Take $r = \delta$ and assume (14) holds, we have

$$c(F, K_c, r, \tau, \psi) = (1 + x) p(F, K_p, r, \tau, \psi), \quad (15)$$

where $K_c = (1 + x)F$ and $K_p = F/(1 + x)$, with $x > 0$.

Empirical Evidence of Symmetry

We restrict to Lévy markets with jump measure of the form

$$\Pi(dy) = e^{\beta y} \Pi_0(dy),$$

where $\Pi_0(dy)$ is a symmetric measure, i.e. $\Pi_0(dy) = \Pi_0(-dy)$, everything with respect to the risk neutral measure \mathbb{Q} .

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As a consequence of (14), market is symmetric if and only if $\beta = -1/2$.

In view of this, we propose to measure the *asymmetry* in the market through the parameter $\beta + 1/2$. When $\beta + 1/2 = 0$ we have a symmetric market.

Esscher Transform

We can obtain an Equivalent Martingale Measure by

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As a consequence:

$$\beta_{\mathbb{Q}} = \beta_{\mathbb{P}} + \theta$$

Example 0: Diffusions with jumps

Consider the jump - diffusion model proposed by Merton (1976). The driving Lévy process in this model has Lévy measure given by

$$\Pi(dy) = \lambda \frac{1}{\delta\sqrt{2\pi}} e^{-(y-\mu)^2/(2\delta^2)} dy,$$

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That result is obtained as a particular case: $\beta = \frac{\mu}{\delta^2}$

Example 1

Consider the Generalized Hyperbolic Distributions, with Lévy measure:

$$\Pi(dy) = e^{\beta y} \frac{1}{|y|} \left(\int_0^\infty \frac{\exp(-\sqrt{2z + \alpha^2|y|})}{\pi^2 z (J_\lambda^2(\delta\sqrt{2z}) + Y_\lambda^2(\delta\sqrt{2z}))} dz + \mathbf{1}_{\{\lambda \geq 0\}} \lambda e^{-\alpha|y|} \right) dy$$

where $\alpha, \beta_{\mathbb{P}}, \lambda, \delta$ are the historical parameters that satisfy the conditions $0 \leq |\beta_{\mathbb{P}}| < \alpha$, and $\delta > 0$; and J_λ, Y_λ are the Bessel functions of the first and second kind.

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$$\beta_{\mathbb{P}} = -0.0035 \quad \text{and} \quad \beta_{\mathbb{Q}} = 80.65.$$

Estimated Parameters GH

<i>Sample</i>	α	β	δ	μ	λ	<i>LLH</i>
Bbas4	30.7740	3.5267	0.0295	-0.0051	-0.0492	3512.73
Bbdc4	47.5455	-0.0006	0	0	1	3984.49
Brdt4	56.4667	3.4417	0.0026	-0.0026	1.4012	3926.68
Cmig4	1.4142	0.7491	0.0515	-0.0004	-2.0600	3685.43
Csna3	46.1510	0.0094	0	0	0.6910	3987.52
Ebtp4	3.4315	3.4316	0.0670	-0.0071	-2.1773	1415.64
Elet6	1.4142	0.0120	0.0524	0	-1.8987	3539.06
lbvsp	1.7102	-0.0035	0.0357	0.0020	-1.8280	4186.31
Itau4	49.9390	1.7495	0	0	1	4084.89
Petr4	7.0668	0.4848	0.0416	0.0003	-1.6241	3767.41
Tcsl4	1.4142	0	0.0861	0.0011	-2.6210	1329.64
Tlpp4	6.8768	0.4905	0.0359	0	-1.3333	3766.28
Tnep4	2.2126	2.2127	0.0786	-0.0028	-2.2980	1323.66
Tnlp4	1.4142	0.0021	0.0590	0.0005	-2.1536	1508.22
Vale5	25.2540	2.6134	0.0265	-0.0015	-0.6274	3958.47

Example 2

Consider the Meixner distribution, with Lévy measure:

$$\Pi(dy) = c \frac{e^{\frac{b}{a}y}}{y \sinh(\pi y/a)} dy,$$

where a , b and c are parameters of the Meixner density, such that $a > 0$, $-\pi < b < \pi$ and $c > 0$. Then $\beta_{\mathbb{P}} = b/a$.

Estimated Parameters Meixner

Index	\hat{a}	\hat{b}	θ	$\beta_0 + 1/2$
Nikkei 225	0.02982825	0.12716244	0.42190524	5.18506
DAX	0.02612297	-0.50801886	-4.46513538	-23.4123
FTSE-100	0.01502403	-0.014336370	-4.34746821	-4.8017
Nasdaq Comp.	0.03346698	-0.49356259	-5.95888693	-20.2066
CAC-40.	0.02539854	-0.23804755	-5.77928595	-14.6518

Schoutens (2002) estimates with data 1/1/1997 to 12/31/1999

Example 3

This CGMY model, proposed by Carr et al. (2002) is characterized by $\sigma = 0$ and Lévy measure given by (1), where the function $p(y)$ is given by

$$p(y) = \frac{C}{|y|^{1+Y}} e^{-\alpha|y|}.$$

The parameters satisfy $C > 0$, $Y < 2$, and $G = \alpha + \beta \geq 0$, $M = \alpha - \beta \geq 0$, where C, G, M, Y are the parameters of the model.

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Values of $\beta = (G - M)/2$ are obtained for different assets under the market risk neutral measure and in the general situation, the parameter β is negative and less than $-1/2$.

Implied volatility

- Any model satisfying (14) must have identical Black-Scholes implicit volatilities for calls and puts with strikes $\ln(K_c/F) = \ln x = -\ln(K_p/F)$, with $x > 0$ arbitrary.

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- That is, the volatility smile curve is symmetric in the moneyness $\ln(K/F)$.
- By put-call parity, European calls and puts with same strike and maturity must have identical implicit volatilities.

Skewness Premium (SK)

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Skewness Premium (SK)

The SK was addressed for the following stochastic processes:

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- Stochastic Volatility processes, the benchmark model being those for which volatility evolves independently of the asset price:

$$\begin{aligned} dS_t &= \mu S_t dt + \sigma_t S_t dB_t^1, \\ d\sigma_t &= \delta(\theta - \sigma_t) dt + \vartheta dB_t^2 \end{aligned}$$

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- Jump-diffusion processes, the benchmark model is the Merton's (1976) model:

$$dS_t = (\mu - \lambda\kappa)S_t dt + \sigma S_t dB_t + S_t dq_t$$

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- $\ln(Y)$ is Normal with variance δ^2 .

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Calls $x\%$ in-the-money should cost $0\% - x\%$ less than puts $x\%$ in-the-money.

Proof

By duality we have:

$$C(F, T; Fk, \sigma k^{1-\rho}, \rho, b) = kP(F, T; F/k, \sigma, \rho, b), \text{ for } k > 0.$$

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The idea is to exploit monotonicity of option prices on volatility.
If $k > 1$ and $\rho < 1$ then $\sigma k^{1-\rho} > \sigma$,

$$C(F, T; Fk, \sigma, \rho, b) < C(F, T; Fk, \sigma k^{1-\rho}, \rho, b)$$

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Main Problems

European option price is monotonically increasing in the volatility if and only if the option price, at each fixed time prior to maturity, is convex as function of the price of the underlying asset.

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Main results

Theorem

Take $r = \delta$ and assume $\Pi(dy) = \lambda F(dy)$, $\lambda > 0$, if F is such that $\int e^y F(dy) \geq 1$, then

$$c(F_0, K_c, r, \tau, \psi) \geq (1 + x) p(F_0, K_p, r, \tau, \psi), \quad (17)$$

where $K_c = (1 + x)F_0$ and $K_p = F_0/(1 + x)$, with $x > 0$.

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Let $\tilde{\lambda} = \lambda \int e^{-y} F(-dy)$ and $\tilde{F}(dy) = \frac{e^{-y} F(-dy)}{\int e^{-y} F(-dy)}$. Then,

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$$\begin{aligned} c(F_0, K_c, r, \tau, a, \sigma, \tilde{\lambda}\tilde{F}) &\geq c(F_0, K_c, r, \tau, a, \sigma, \lambda F) \\ &= (1+x)p(F_0, K_p, r, \tau, a, \sigma, \tilde{\lambda}\tilde{F}) \end{aligned}$$

where the last equality is obtained from duality.

Main results

Theorem

Take $r = \delta$ and assume $\Pi(dy) = e^{\beta y} \Pi_0(dy)$, if $\beta \geq -1/2$, then

$$C(F, K_c, r, \tau, \psi) \geq (1 + x) P(F, K_p, r, \tau, \psi), \quad (18)$$

where $K_c = (1 + x)F$ and $K_p = F/(1 + x)$, with $x > 0$.

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By monotonicity

$$\begin{aligned} c(F_0, K_C, r, \delta, \tau, a, \sigma, \Pi) &\geq c(F_0, K_C, r, \tau, a, \sigma, \tilde{\Pi}) \\ &= (1+x)p(F_0, K_C, r, \tau, a, \sigma, \Pi), \end{aligned}$$

where the last equality is obtained from duality and the fact that $\tilde{\tilde{\Pi}} = \Pi$.

Merton Model

Remember that $\beta = \frac{\mu}{\delta^2}$, so we obtain that sufficient conditions in the above theorems are equivalent. That is,

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$$\int e^{-y} F(-dy) = e^{\mu + \delta^2/2} \geq 1 \iff \beta \geq -1/2.$$

In general it is not true.

Conclusions

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- Skewness Premium: Call option $x\%$ OTM should be priced $[0, x\%]$ more than Put options $x\%$ OTM. ($SK \geq 0?$)
- The SK can not identify which process or which parameter values best fit observed option data.
- Which of the Lévy processes and associated option pricing models can generate the observed moneyness biases.

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Gyrsanov for Semimartingales

Lemma

Let Y be a d -dimensional semimartingale with finite variation with triplet (B, C, ν) under P , let u, v be vectors in \mathbb{R}^d and $v \in [-M, M]^d$. Moreover let $\tilde{P} \sim P$, with density $\frac{d\tilde{P}}{dP} = \frac{e^{\langle v, Y_T \rangle}}{\mathbb{E}[e^{\langle v, Y_T \rangle}]}$. Then the process $Y^* := \langle u, Y \rangle$ is a \tilde{P} -semimartingale with triplet:

$$b_s^* = \langle u, b_s \rangle + \frac{1}{2} (\langle u, c_s v \rangle + \langle v, c_s u \rangle) + \int_{\mathbb{R}^d} \langle u, x \rangle (e^{\langle v, x \rangle} - 1) \lambda_s(dx)$$

$$c_s^* = \langle u, c_s u \rangle$$

$$\lambda_s^* = \Lambda(\kappa_s)$$

where Λ is a mapping $\lambda : \mathbb{R}^d \rightarrow \mathbb{R}$ such that $x \mapsto \Lambda(x) = \langle u, x \rangle$ and κ_s is a **measure** defined by $\kappa_s(A) = \int_A e^{\langle v, x \rangle} \lambda_s(dx)$.