Mechanics of Options Markets

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Review of Option Types

- A call is an option to buy
- A put is an option to sell
- A European option can be exercised only at the end of its life
- An American option can be exercised at any time

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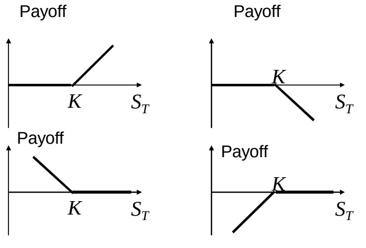
Payoffs from Options
What is the Option Position in Each Case?

K = Strike price, S_T = Price of asset at maturity



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Assets Underlying Exchange-Traded Options

- Stocks: VALE5,PETR4,OGX
- Foreign Currency
- Stock Indices

SP500(SPX), Ibovespa (IBV),Euro stoxx50 (FESX)

- Futures
- BTC

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Specification of Exchange-Traded Options

- Expiration date
- Strike price
- European or American
- Call or Put (option class)

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Terminology

Moneyness :

- At-the-money option
- In-the-money option
- Out-of-the-money option

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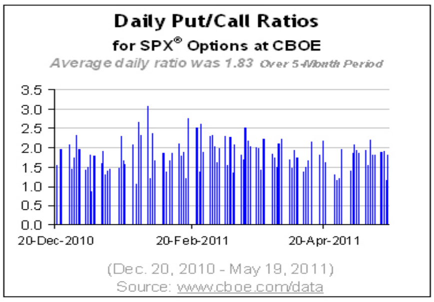
Puts vs Calls



Properties of Stock Options

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Notation

- c: European call option price
- p: European put option price
- S_0: Stock price today
- K: Strike price
- T: Life of option
- σ: Volatility of stock price
- C: American call option price
- P: American put option price
- S_T: Stock price at option maturity
- D: PV of dividends paid during life of option
- r Risk-free rate for maturity T with cont. comp.

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Effect of Variables on Option Pricing

Variable	c	p	C	P
S_0	+	−	+	−
K	−	+	−	+
T	?	?	+	+
σ	+	+	+	+
r	+	−	+	−
D	−	+	−	+

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American vs European Options

An American option is worth at least as much as the corresponding European option

$$C \geq c$$
$$P \geq p$$

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Calls: An Arbitrage Opportunity?

- Suppose that

c = 3	S_0 = 20
T = 1	r = 10%
K = 18	D = 0

- Is there an arbitrage opportunity?

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Lower Bound for European Call Option Prices; No Dividends

$$c \geq S_0 - Ke^{-rT}$$

Puts: An Arbitrage Opportunity?

- Suppose that

$p = 1$	$S_0 = 37$
$T = 0.5$	$r = 5\%$
$K = 40$	$D = 0$

- Is there an arbitrage opportunity?

Lower Bound for European Put Prices; No Dividends

$$p \geq Ke^{-rT} - S_0$$

Put-Call Parity: No Dividends

- Consider the following 2 portfolios:
  - Portfolio A: European call on a stock + zero-coupon bond that pays K at time T
  - Portfolio B: European put on the stock + the stock

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Values of Portfolios

		$S_T > K$	$S_T < K$
Portfolio A	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio B	Put Option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

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The Put-Call Parity Result

- Both are worth max(S_T , K) at the maturity of the options
- They must therefore be worth the same today. This means that

$$c + Ke^{-rT} = p + S_0$$

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Arbitrage Opportunities

- Suppose that
  $c = 3$ $S_0 = 31$
  $T = 0.25$ $r = 10\%$
  $K = 30$ $D = 0$
- What are the arbitrage possibilities when
  $p = 2.25$?
  $p = 1$?

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Early Exercise

- Usually there is some chance that an American option will be exercised early
- An exception is an American call on a non-dividend paying stock
- This should never be exercised early

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An Extreme Situation

- For an American call option:
  $S_0 = 100$; $T = 0.25$; $K = 60$; $D = 0$
  Should you exercise immediately?
- What should you do if
  - You want to hold the stock for the next 3 months?
  - You do not feel that the stock is worth holding for the next 3 months?

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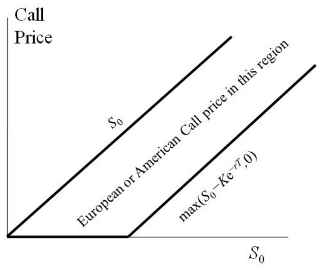
Reasons For Not Exercising a Call Early (No Dividends)

- No income is sacrificed
- You delay paying the strike price
- Holding the call provides insurance against stock price falling below strike price

Bounds for European or American Call Options (No Dividends)



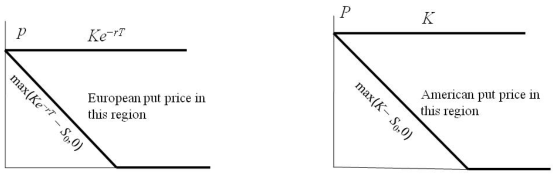
Should Puts Be Exercised Early ?

Are there any advantages to exercising an American put when

S_0 = 60; T = 0.25; r=10%

K = 100; D = 0

Bounds for European and American Put Options (No Dividends)



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The Impact of Dividends on Lower Bounds to Option Prices

$$c \geq S_0 - D - Ke^{-rT}$$
$$p \geq D + Ke^{-rT} - S_0$$

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Extensions of Put-Call Parity

- American options; $D = 0$
  $S_0 - K < C - P < S_0 - Ke^{-rT}$
- European options; $D > 0$
  $c + D + Ke^{-rT} = p + S_0$
- American options; $D > 0$
  $S_0 - D - K < C - P < S_0 - Ke^{-rT}$

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Put-Call Duality

Fajarado and Mordecki (2006): For calls and puts in the same underlying American or Europeans

$$\mathrm{C}(S_0, K, r, \delta, T, \sigma) = \mathrm{P}(K, S_0, \delta, r, T, \sigma)$$

With Jumps

$$\mathrm{C}(S_0, K, r, \delta, T, \pi) = \mathrm{P}(K, S_0, \delta, r, T, \pi)$$

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