The Greek Letters

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1

Example

- A bank has sold for $300,000 a European call option on 100,000 shares of a non-dividend paying stock
- S_0 = 49, K = 50, r = 5%, σ = 20%, T = 20 weeks, μ = 13%
- The Black-Scholes value of the option is $240,000
- How does the bank hedge its risk to lock in a $60,000 profit?

2

Naked & Covered Positions

Naked position
Take no action
Covered position
Buy 100,000 shares today

Both strategies leave the bank exposed to significant risk

3

Stop-Loss Strategy

This involves:

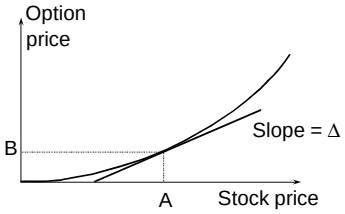
- Buying 100,000 shares as soon as price reaches $50
- Selling 100,000 shares as soon as price falls below $50

This deceptively simple hedging strategy does not work well

4

Delta

- Delta (Δ) is the rate of change of the option price with respect to the underlying



5

Delta Hedging

- This involves maintaining a delta neutral portfolio
- The delta of a European call on a non-dividend paying stock is $N(d_1)$
- The delta of a European put on the stock is

$$N(d_1) - 1$$

6

Delta Hedging continued

- The hedge position must be frequently rebalanced
- Delta hedging a written option involves a "buy high, sell low" trading rule

7

Theta

- Theta (Θ) of a derivative (or portfolio of derivatives) is the rate of change of the value with respect to the passage of time
- The theta of a call or put is usually negative. This means that, if time passes with the price of the underlying asset and its volatility remaining the same, the value of a long option declines

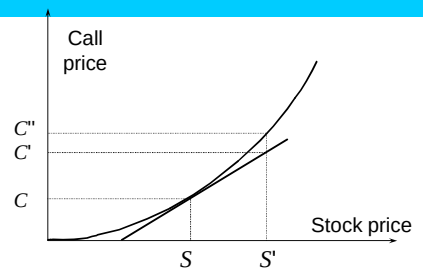
8

Gamma

- Gamma (Γ) is the rate of change of delta (Δ) with respect to the price of the underlying asset
- Gamma is greatest for options that are close to the money (see Figure 17.9, page 364)

9

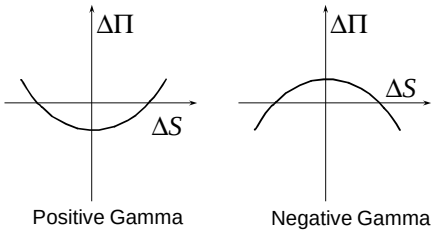
Gamma Addresses Delta Hedging Errors Caused By Curvature



10

Interpretation of Gamma

For a delta neutral portfolio, $\Delta\Pi \approx \Theta\,\Delta t + \frac{1}{2}\Gamma\Delta S^2$



11

Relationship Between Delta, Gamma, and Theta

For a portfolio of derivatives on a stock paying a continuous dividend yield at rate q

$$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2\Gamma = r\Pi$$

12

Vega

- Vega ($\mathcal{V}$) is the rate of change of the value of a derivatives portfolio with respect to volatility

13

Managing Delta, Gamma, & Vega

- Δ can be changed by taking a position in the underlying
- To adjust Γ & ν it is necessary to take a position in an option or other derivative

14

Rho

- Rho is the rate of change of the value of a derivative with respect to the interest rate
- For currency options there are 2 rhos

15

Hedging in Practice

- Traders usually ensure that their portfolios are delta-neutral at least once a day
- Whenever the opportunity arises, they improve gamma and vega
- As portfolio becomes larger hedging becomes less expensive

16

Hedging vs Creation of an Option Synthetically

- When we are hedging we take positions that offset Δ, Γ, $\mathcal{V}$, etc.
- When we create an option synthetically we take positions that match Δ, Γ, & $\mathcal{V}$

17