

Volatility Smiles

Prf. José Fajardo
Fundação getulio Vargas

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What is a Volatility Smile?

- ✦ It is the relationship between implied volatility and strike price for options with a certain maturity
- ✦ The volatility smile for European call options should be exactly the same as that for European put options
- ✦ The same is at least approximately true for American options

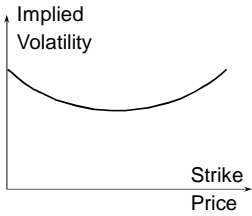
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Why the Volatility Smile is the Same for Calls and Put

- ✦ Put-call parity $p + S_0 e^{-qT} = c + K e^{-rT}$ holds for market prices (p_{mkt} and c_{mkt}) and for Black-Scholes prices (p_{bs} and c_{bs})
- ✦ It follows that $p_{mkt} - p_{bs} = c_{mkt} - c_{bs}$
- ✦ When $p_{bs} = p_{mkt}$, it must be true that $c_{bs} = c_{mkt}$
- ✦ It follows that the implied volatility calculated from a European call option should be the same as that calculated from a European put option when both have the same strike price and maturity

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The Volatility Smile for Foreign Currency Options



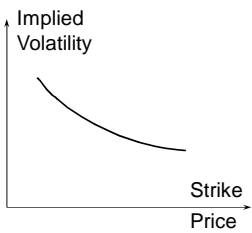
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Implied Distribution for Foreign Currency Options

- ✦ Both tails are heavier than the lognormal distribution
- ✦ It is also "more peaked" than the lognormal distribution

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The Volatility Smile for Equity Options



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Implied Distribution for Equity Options

- The left tail is heavier and the right tail is less heavy than the lognormal distribution

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Other Volatility Smiles?

What is the volatility smile if

- True distribution has a less heavy left tail and heavier right tail
- True distribution has both a less heavy left tail and a less heavy right tail

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Ways of Characterizing the Volatility Smiles

- Plot implied volatility against K/S_0 (The volatility smile is then more stable)
- Plot implied volatility against K/F_0 (Traders usually define an option as at-the-money when K equals the forward price, F_0 , not when it equals the spot price S_0)
- Plot implied volatility against delta of the option (This approach allows the volatility smile to be applied to some non-standard options. At-the money is defined as a call with a delta of 0.5 or a put with a delta of -0.5 .)

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Possible Causes of Volatility Smile

- ✦ Asset price exhibits jumps rather than continuous changes
- ✦ Volatility for asset price is stochastic
 - ▣ In the case of an exchange rate volatility is not heavily correlated with the exchange rate. The effect of a stochastic volatility is to create a symmetrical smile
 - ▣ In the case of equities volatility is negatively related to stock prices because of the impact of leverage. This is consistent with the skew that is observed in practice

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Volatility Term Structure

- ✦ In addition to calculating a volatility smile, traders also calculate a volatility term structure
- ✦ This shows the variation of implied volatility with the time to maturity of the option

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Volatility Term Structure

The volatility term structure tends to be downward sloping when volatility is high and upward sloping when it is low

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Example of a Volatility Surface

	K/S_0				
	0.90	0.95	1.00	1.05	1.10
1 mnt	14.2	13.0	12.0	13.1	14.5
3 mnt	14.0	13.0	12.0	13.1	14.2
6 mnt	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0

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Greek Letters

- ✦ If the Black-Scholes price, c_{BS} is expressed as a function of the stock price, S , and the implied volatility, σ_{imp} , the delta of a call is

$$\frac{\partial c_{BS}}{\partial S} + \frac{\partial c_{BS}}{\partial \sigma_{imp}} \frac{\partial \sigma_{imp}}{\partial S}$$

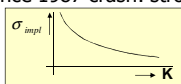
- ✦ Is the delta higher or lower than

$$\frac{\partial c_{BS}}{\partial S}$$

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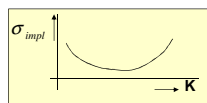
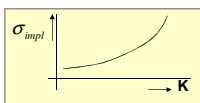
Market Skews

Dominating fact since 1987 crash: strong negative skew on Equity Markets



Not a general phenomenon

Gold:



We focus on Equity Markets.

What would be the shape of imp vol in BTC options?

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BTC Options Imp Vol

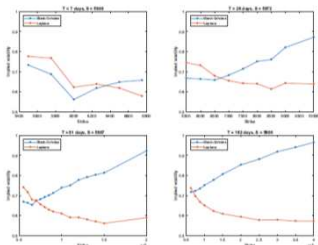


Fig. 4 Implied volatility smiles corresponding to out of the money options on 29 June 2018

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Skews

- ✦ Volatility Skew: slope of implied volatility as a function of Strike
- ✦ Link with Skewness (asymmetry) of the Risk Neutral density function ϕ ?

Moments	Statistics	Finance
1	Expectation	FWD price
2	Variance	Level of implied vol
3	Skewness	Slope of implied vol
4	Kurtosis	Convexity of implied vol

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Why Volatility Skews?

- ✦ Market prices governed by
 - ▣ a) Anticipated dynamics (future behavior of volatility or jumps)
 - ▣ b) Supply and Demand

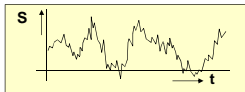


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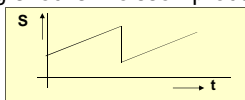
Modeling Uncertainty

Main ingredients for spot modeling

- ✦ Many small shocks: Brownian Motion (continuous prices)



- ✦ A few big shocks: Poisson process (jumps)

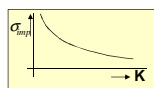


- ✦ Or Many big shocks: Lévy Process (GH)

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Mechanisms to produce Skews (1)

- ✦ To obtain downward sloping implied volatilities

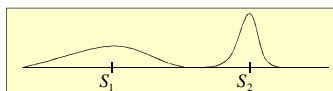


- ❑ a) Negative link between prices and volatility
- ❑ b) Downward jumps

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Mechanisms to produce Skews (2)

- ❑ a) Negative link between prices and volatility



- ❑ b) Downward jumps

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Extracting information

MEXBOL: Option Prices

ISZ7 129100.00 -648.00 29120.00/29313.00 2x10 Index OMON
 At DELAYED Vol 908 Op 29015.00 HI 29500.00 Lo 29015.00 OpInt 41.141

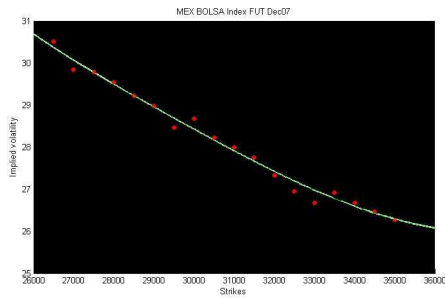
Option Monitor: MEX BOLSA IDX FUT Dec07
 Center 29100.00 Number of Strikes 18 -or- from Center Exchange C (Composite)

ISZ7 DEC 07	Strike (Contract Size 10)	Bid	Ask	RTB	RTA	ISZ7 DEC 07	Strike (Contract Size 10)	Bid	Ask	RTB	RTA	
1	13270	2580				10	13270	2580				
2	13270	2580				11	13270	2580				
3	13270	2580				12	13270	2580				
4	13270	2774.00	2862.00	35.77	39.76	13	13270	2650	183.00	377.00	25.59	
5	13270	2780	2273.00	2843.00	26.25	37.83	14	13270	2700	184.00	237.00	25.22
6	13270	2780	1966.00	2145.00	23.72	36.33	15	13270	2750	301.00	414.00	27.55
7	13270	2880	1577.00	1783.00	20.55	35.05	16	13270	2800	438.00	554.00	24.97
8	13270	2880	1243.00	1454.00	17.42	34.61	17	13270	2850	607.00	751.00	25.24
9	13270	2980	923.00	1160.00	14.30	33.88	18	13270	2900	833.00	977.00	25.46
10	13270	2980	583.00	817.00	11.01	32.64	19	13270	2950	1064.00	1233.00	25.14
11	13270	3080	438.00	606.00	12.33	31.68	20	13270	3000	1273.00	1537.00	23.79
12	13270	3080	346.00	522.00	12.28	31.32	21	13270	3050	1572.00	1972.00	18.41
13	13270	3180	283.00	378.00	14.05	30.74	22	13270	3100	1946.00	2240.00	16.15
14	13270	3180	187.00	264.00	16.29	30.12	23	13270	3150	2314.00	2674.00	13.23
15	13270	3280	74.00	173.00	23.18	29.68	24	13270	3200	2743.00	3076.00	10.51
16	13270	3280	42.00	120.00	28.36	29.29	25	13270	3250	3212.00	3514.00	10.23
17	13270	3380	22.00	77.00	35.32	28.89	26	13270	3300	3689.00	3952.00	11.46
18	13270	3380	10.00	47.00	32.62	28.43	27	13270	3350	4185.00	4411.00	13.78

Australia 61 2 9377 0600 Brazil 5511 3048 4500 Europe 44 20 7320 7500 Germany 49 49 300413 Hong Kong 852 2397 6000
 Japan 81 3 2001 8900 Singapore 65 6212 1000 U.S. 1 212 218 2000 copyright 1998-2018 Bloomberg Finance L.P.
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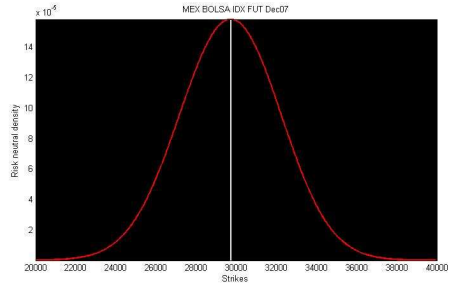
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Non parametric fit of implied vols



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Risk Neutral density



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S&P 500: Option Prices

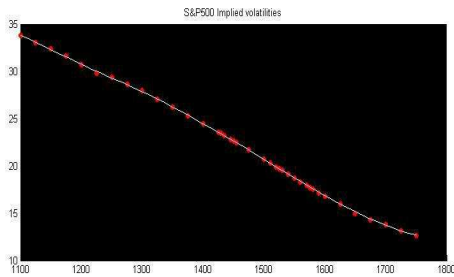
SPX ↑ 1436.96 +3.69 Index OMON
 At 12:38 Op 1434.51 Hi 1452.04 Lo 1433.42 Prev 1433.27
 Option Monitor: S&P 500 INDEX
 Center 1430.51 Number of Strikes 18 -or- from Center Exchange C (Composite)

Ticker	Strike	Bid	Ask	RTB	RTA	Ticker	Strike	Bid	Ask	RTB	RTA
SPX	1436.96					SPX	1436.96				
1) SPX145	1385	70.60	72.60	28.02	23.30	10) SPX145	1435	21.90	24.40	24.10	25.00
2) SPX140	1400	55.90	58.00	27.65	23.50	11) SPX140	1400	24.80	25.40	24.41	25.05
3) SPX145	1405	53.00	55.00	27.27	23.50	12) SPX145	1405	24.60	25.60	23.50	24.07
4) SPX140	1410	53.70	51.70	26.88	23.11	13) SPX140	1410	26.30	26.40	23.48	24.50
5) SPX145	1415	55.20	58.00	26.47	27.70	14) SPX145	1415	27.50	23.50	22.51	24.15
6) SPX140	1420	52.00	54.00	26.05	27.00	15) SPX140	1420	29.10	31.10	22.50	23.01
7) SPX145	1425	49.00	51.50	25.79	26.90	16) SPX145	1425	30.70	31.50	22.20	22.68
8) SPX140	1430	46.20	48.20	25.32	26.50	17) SPX140	1430	32.40	34.40	21.91	23.72
9) SPX145	1435	43.10	45.10	24.97	26.10	18) SPX145	1435	30.20	30.20	21.48	22.43
10) SPX140	1440	40.00	42.00	24.57	25.77	19) SPX140	1440	35.10	35.00	21.45	22.10
11) SPX145	1445	37.00	39.00	24.17	25.37	20) SPX145	1445	35.10	40.10	20.95	21.85
12) SPX140	1450	34.10	36.10	23.77	24.90	21) SPX140	1450	40.20	41.50	20.25	21.05
13) SPX145	1455	31.30	33.40	23.37	24.64	22) SPX145	1455	42.40	44.40	19.68	21.45
14) SPX140	1460	28.70	30.70	23.03	24.25	23) SPX140	1460	41.70	45.70	19.40	20.53
15) SPX145	1465	26.10	27.80	22.61	23.66	24) SPX145	1465	42.10	45.20	18.94	20.25
16) SPX140	1470	23.70	25.70	22.25	23.51	25) SPX140	1470	49.70	51.70	18.51	19.70
17) SPX145	1475	21.30	23.30	21.82	23.10	26) SPX145	1475	52.30	54.20	17.99	19.31
18) SPX140	1480	19.30	20.30	21.52	22.61	27) SPX140	1480	55.10	52.10	17.45	18.85

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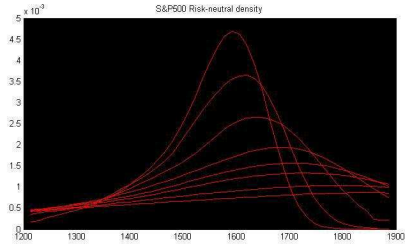
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Non parametric fit of implied vols



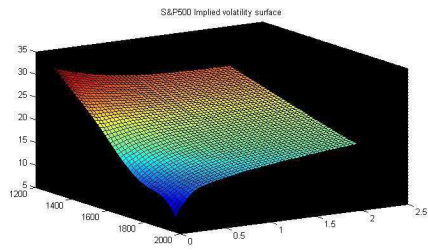
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Risk Neutral densities



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Implied Volatilities



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