Multifactor Models

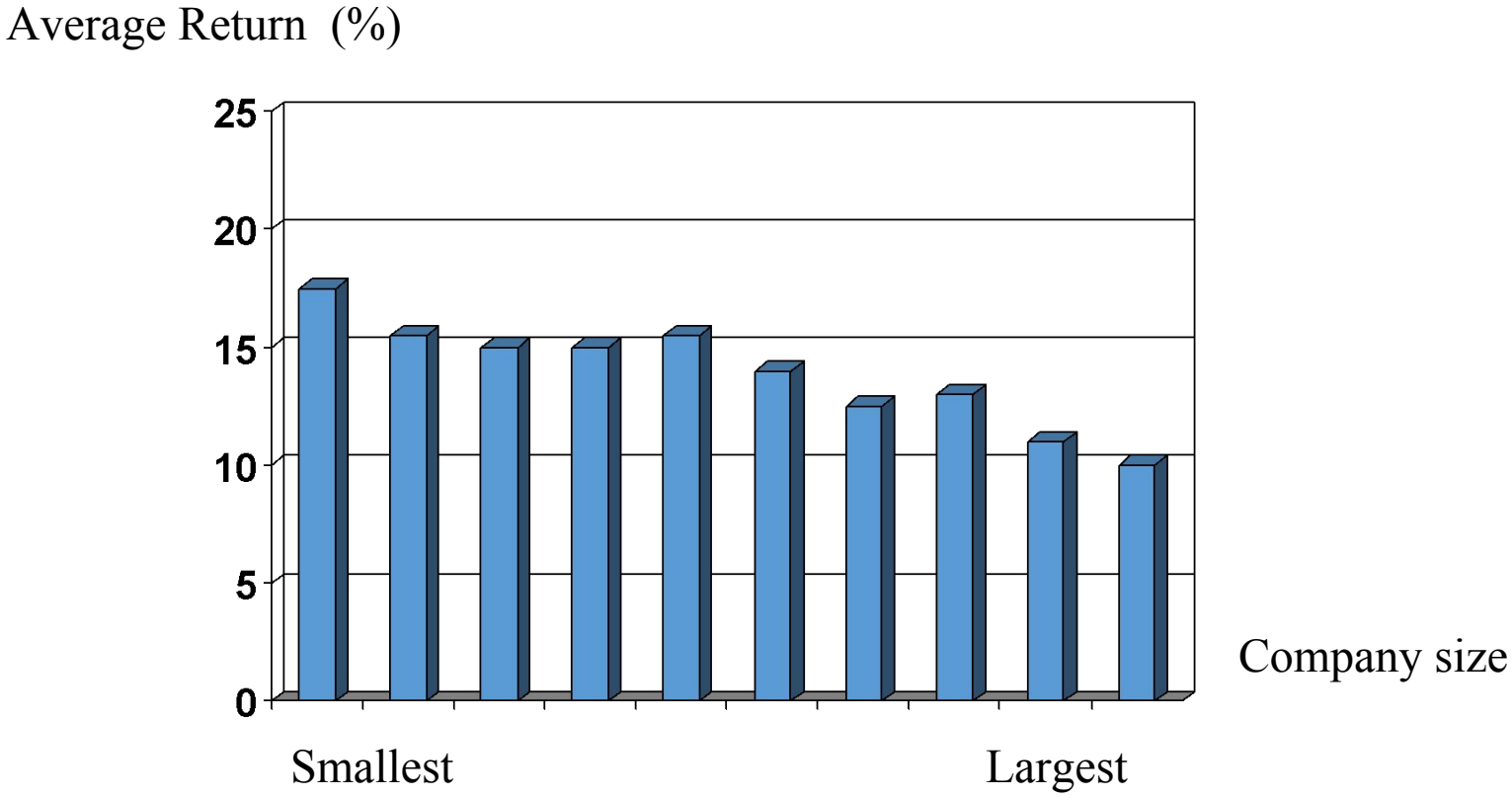
- Other factors can be added
  - Inflation, Industrial Production, etc
  - Firm specific: Book/Market, dividend/price, etc.

- Fama & French (*J. Finance* 1993,427-466)
  - book-to-market and size

$$r_i - r_f = ß_{im}(r_m - r_f) + ß_{is}(r_s - r_b) + ß_{il}(r_h - r_l)$$

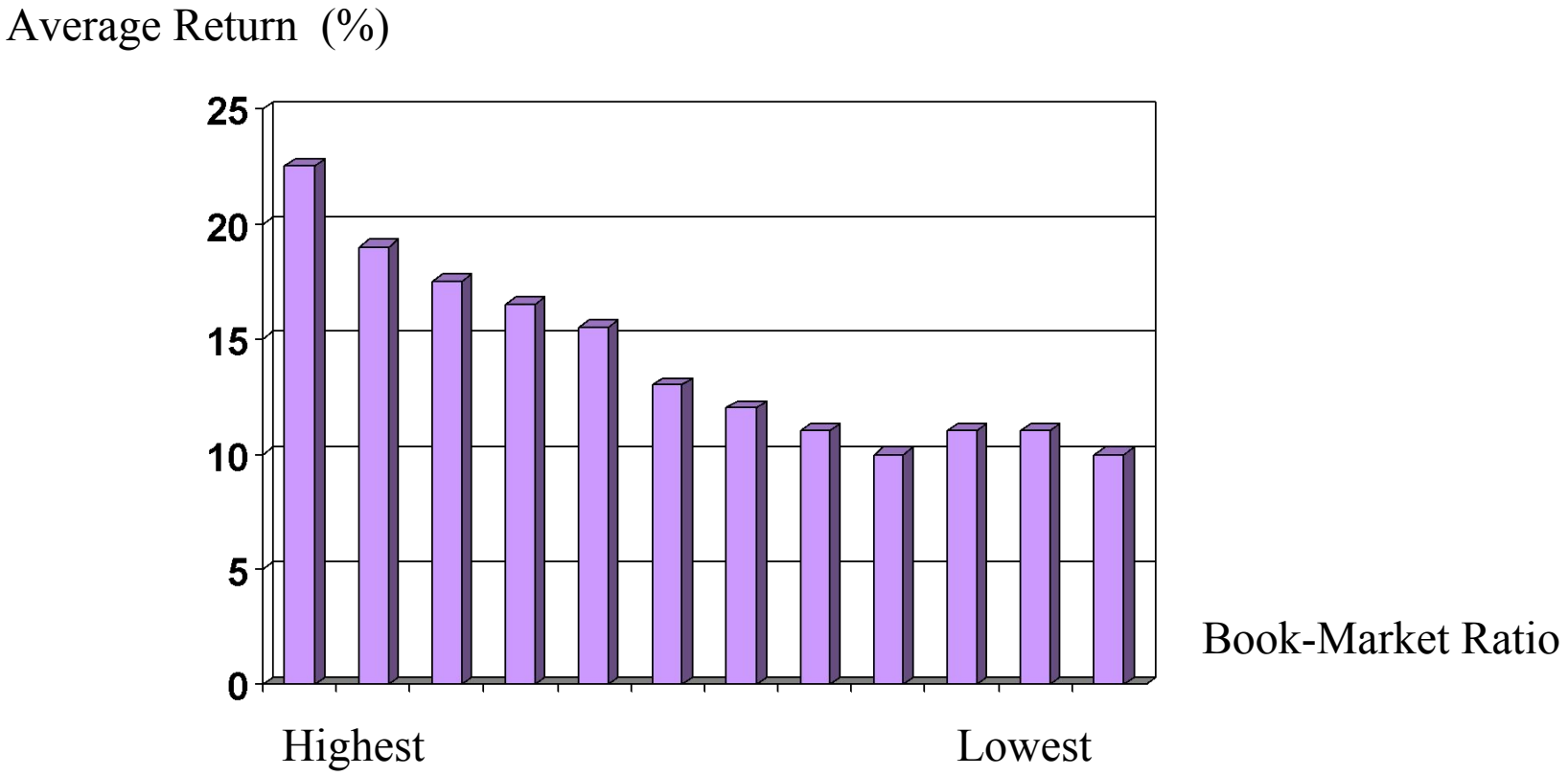
Multifactor Models

Company Size vs. Average Return



Modelo Multifatores

Book-Market vs. Average Return



Modelo Multifatores

Elton, Gruber e Mei (1994)

(1978-1990)

Factor	Estimated Risk Premium $(r_{factor} - r_f)$
Yield spread	5.10%
Interest rate	-.61
Exchange rate	-.59
Real GNP	.49
Inflation	-.83
Mrket	6.36

CAPM extensions

- Black (1972):
- There is no risk-free asset
- Given two efficient portfolios P and Q the expected return of any asset can be expressed as:

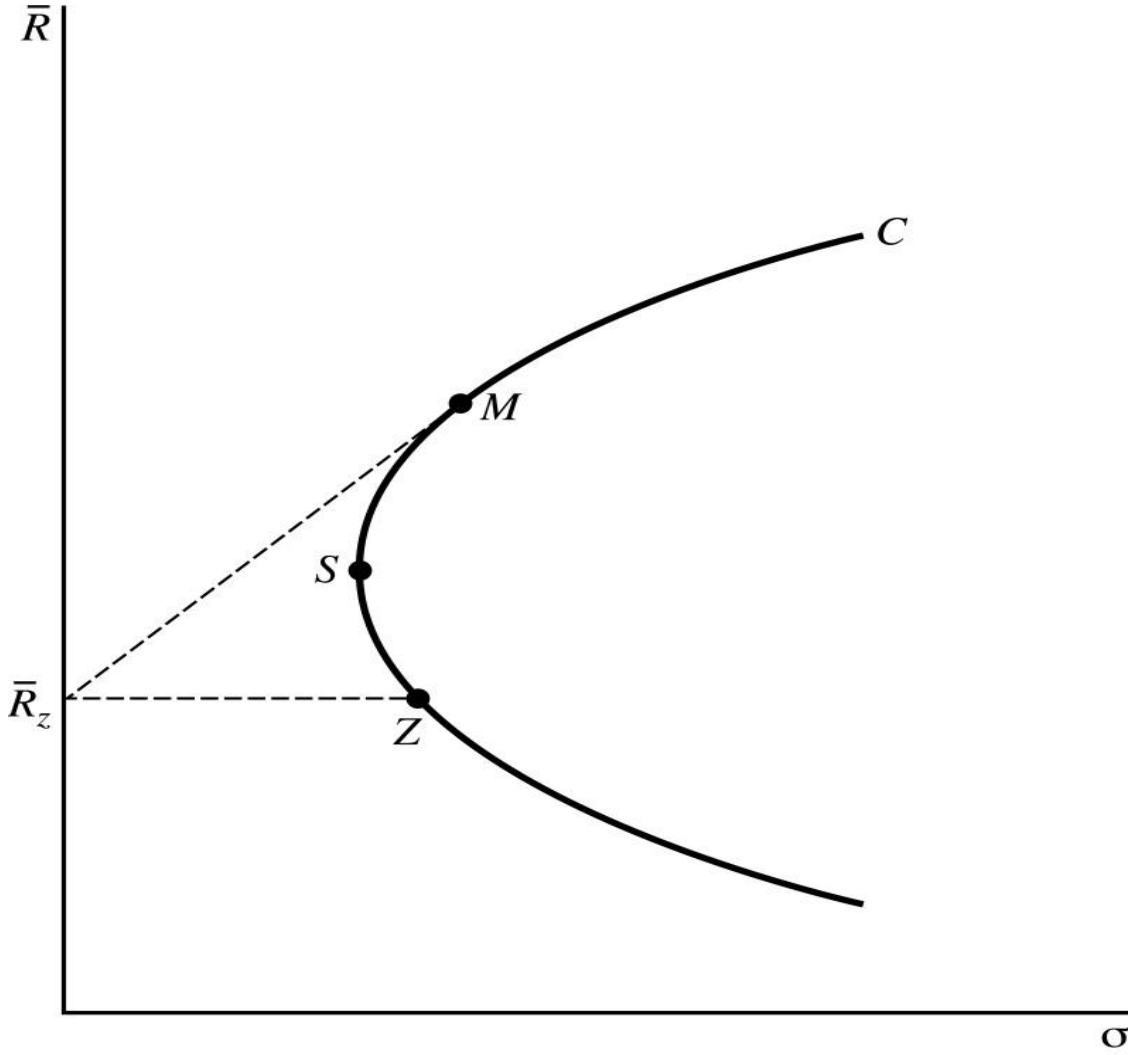
$$E(r_i) = E(r_Q) + \left[E(r_P) - E(r_Q) \right] \frac{Cov(r_i, r_P) - Cov(r_P, r_Q)}{\sigma^2{}_P - Cov(r_P, r_Q)}$$

CAPM extensions

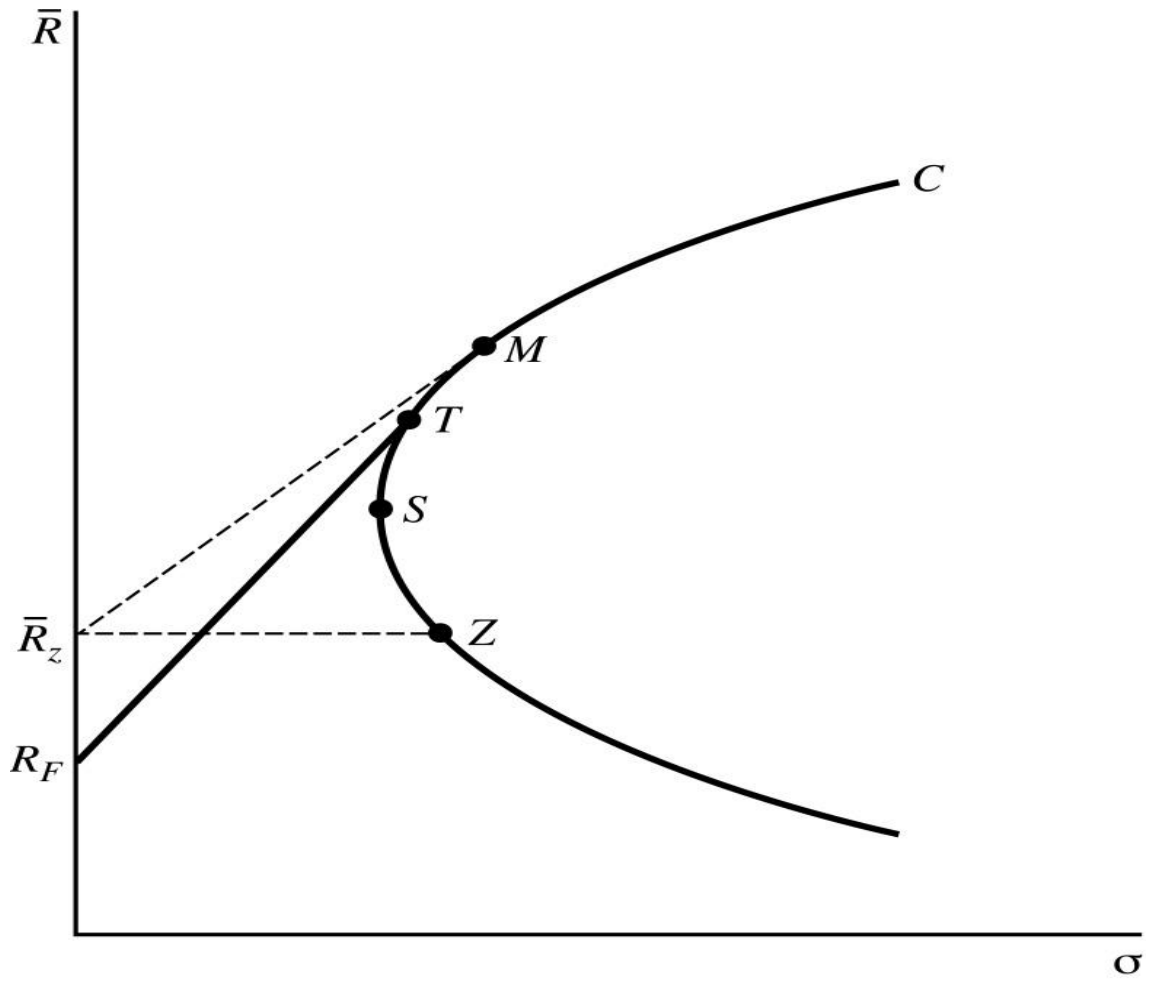
- It is possible to find a portfolio in the inferior par of the frontier *Z(P)* (not efficient) for all portfolio *P* in the efficient part, such that: $Cov(r_P, r_{Z(P)})=0$.
- It is called *Zero Beta,* then the expected return of any asset will be given by:

$$E(r_i) = E(r_{Z(M)}) + \left[E(r_M) - E(r_{Z(M)})\right]\frac{Cov(r_i, r_M)}{\sigma^2_M}$$

CAPM extensions



CAPM extensions



CAPM extensions