Short Revision of Econometrics I

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Random Variables

- Probability Space $(\Omega, \mathcal{F}, P)$, where Ω:

$$\Omega = \{0, 1\}, \Omega = \{1, 2, 3, 4, 5, 6\}, ...$$

- Random Variable, is a map: $\Omega \to \mathbf{R}$,

$$X(\{1\}) = 1000, \text{ and } X(\{0\}) = 0$$

Random Variables

- Discrete
- Continuous
- Mixed

Probability Distributions

- Bernoulli(p), Poisson(λ), $\rightarrow P$
- $U(0,1)$, Normal(0,1)$\rightarrow f$

Moments

- Mean
- Variances
- Skewness
- Kurtosis Generalized hyperbolic distributions

Linear Relationship

We have now

$$Y = a + b * X,$$

where a is your fixed salary and $b * X$ is your bonus salary (random), b is a bonus rate. Compute :

- Mean and Variance

If this year you know $a = 700$ and $b = 2\%$. But X has not yet been released, but $X \sim N(\mu_X, \sigma_X^2)$. What are your expectations for your salary?

Two or More Random Variables

In the presence of two or more random variables, we need to compute the joint distribution , in the case of two we denote it by F_{XY}

- $$Covariance(X, Y)$$
- $$Correlation(X, Y)$$
- Variance-Covariance Matrix

Independence

Two events A and B are independent if

$$P(A \cap B) = P(A)P(B).$$

One collection of events $(A_i)_{i \in I}$ is an independent collection of events if for any finite subset J of I, we have

$$P(\bigcap_{j \in J} A_j) = \prod_{j \in J} P(A_j)$$

Independence

Example

Let $\Omega = \{1, 2, 3, 4\}$ and $A = P(\Omega)$.
Let $P(i) = \frac{1}{4},\ i = 1, .., 4$.
Let $A = \{1, 2\},\ B = \{1, 3\},\ C = \{2, 3\}$,
Then A, B, C are independent pairwise but they are not independent

Conditional Probability

Let A, B two events and $P(B) > 0$. The conditional probability of A given B is $P(A/B) = P(A \cap B)/P(B)$.

Suppose $P(B) > 0$

- A and B are independents if and only if $P(A/B) = P(A)$
- The operation $A \to P(A/B)$ from $A \to [0, 1]$ define a new probability measure in A, called conditional probability measure given B

Conditional Probability

(Bayes' Theorem Let (E_n) be a finite or countable partition of Ω, and suppose $P(A) > 0$. Then,

$$P(E_n/A) = \frac{P(A/E_n)P(E_n)}{\sum_m P(A/E_m)P(E_m)}$$

Here Finished class 1