

Derivatives
Problem Set 2
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- 1) Assume that options are written in terms of Bitcoin, have the same maturity of 6 weeks and are ATM . Also, consider that stock price is currently \$9450,00 and the risk-free interest rate is 1.25% p.y. with continuous compounding.
 - a) Draw a diagram showing the variation of investor's profits and losses with the terminal range of stock price for the following portfolio: Long in four shares, long in two puts and short in two calls
 - b) What must be the value of 1(a) portfolio in order to avoid arbitrage opportunities?

- 2) A stock price is currently \$50. Over each of the next two 4-months periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 1.25% per annum with continuous compounding.
 - a) What is the value of a 8-month European call option with strike price of \$52?
 - b) What is the price of a 8-month American put option with strike price \$52? Would it be optimal to exercise it early at any of the nodes of the tree?

- 3) The spot price of BTC is R\$10535.96. In the next two periods of 3 weeks, it can go up 75% or down 68%. The risk-neutral interest rate is 0.25% per annum with continuous compounding.
- What would be the price of European call at the money written in terms of BTC?
 - What would be the price of this call if it be of American kind?
- 4) You were recently hired by an investment bank in NY and them ask you to price an European put in Nasdaq 100, with strike price 7800 and maturity 6 months. The index today's price is 7701.40. You can assume that firms in Nasdaq 100 will not pay dividends in the next 6 months. The risk-neutral interest rate is 1% per annum with continuous compounding.
- You first use a Binomial model with 2 periods and constant volatility of 28%. What price do you obtain?
 - Now consider volatility changes over time. If the first movement of the price in the tree is up, volatility for the next 2nd period will be 20% , But if the first movement is down, volatility will be 40% for the next 2nd period. Volatility remains constant 28% in the 1st period. What is the price of the Put?
- 5) Consider the following contract with a 1 year maturity, it protects 90% of the spot price of Petrobras (PETR4), which is R\$20 but it demands in exchange 20% of profit participation. Now Denote by S_T the PETR4 price at the end of the 1 year period, assume that it satisfy the log-normality assumption. Then we can express the contract payoff as:

$$f(S_T) = \begin{cases} 18, & \text{if } S(T) \leq 20 \\ 20 + 0.8 * (S_T - 20), & \text{otherwise} \end{cases}$$

Assuming that Petrobras volatility is 30%, the risk free interest rate is 7.25% p.y. with continuous compounding. Also, assume that Petrobras will not pay any dividend in that period. Find the price of this contract using risk neutral valuation.

Formulas

- Parity: $c + Xe^{-rT} = p + S_0$

- Binomial Model:

$$f = e^{-rT} [pf_u + (1 - p)f_d],$$

where $p = \frac{e^{rT} - d}{u - d}$

- Log-Normality: $\ln(S_T) - \ln(S_0) \sim Normal\left((\mu - \sigma^2/2)T, \sigma\sqrt{T}\right)$

- Black et Scholes:

$$c = S_0N(d_1) - Ke^{-rT}N(d_1 - \sigma\sqrt{T})$$

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$