

2nd Problem Set—Options
FGV/EBAPE
Asset Pricing

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1. Explain why an American option is always worth at least as much as its intrinsic value.
2. A 1-month European put option on a non-dividend-paying stock is currently selling for \$ 2.50. The stock price is \$47, the strike price is \$50, and the risk-free interest rate is 6% per annum. What opportunities are there for an arbitrageur?
3. Suppose that c_1 , c_2 , and c_3 are the prices of European call options with strike prices K_1 , K_2 , and K_3 , respectively, where $K_3 > K_2 > K_1$ and $K_3 - K_2 = K_2 - K_1$. All options have the same maturity. Show that $c_2 \leq 0.5(c_1 + c_3)$.
4. A stock price is currently \$25. It is known that at the end of 2 months it will be either \$23 or \$27. The risk-free interest rate is 10% per annum with continuous compounding. Suppose S_T is the stock price at the end of 2 months. What is the value of a derivative that pays off S_T^2 at this time?
5. The volatility of a non-dividend-paying stock whose price is \$78, is 30%. The risk-free rate is 3% per annum (continuously compounded) for all maturities. Calculate values for u , d , and p when a 2-month time step is used. What is the value a 4-month European call option with a strike price of \$80 given by a two-step binomial tree. Suppose a trader sells 1,000 options (10 contracts). What position in the stock is necessary to hedge the trader's position at the time of the trade?
6. A stock index is currently 990, the risk-free rate is 5%, and the dividend yield on the index is 2%. Use a three-step tree to value an 18-month American put option with a strike price of 1,000 when the volatility is 20% per annum. How much does the option holder gain by being able to exercise early? When is the gain made?
7. What is *implied volatility* ? How can it be calculated?