

3rd Problem Set (BSM Model-VaR-GARCH)

FGV/EBAPE

Asset Pricing

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1. Consider the following contract with a 1 year maturity, it protects 90% of the spot price of IBM which is \$140 but it demands in exchange 20% of profit participation. Denote by S_T the IBM stock price at the end of the maturity of 1 year. Then we can express the contract payoff as:

$$f(S_T) = \begin{cases} 126, & \text{if } S(T) \leq 126 \\ S_T, & \text{if } 126 < S(T) \leq 140 \\ 140 + 0.8 * (S_T - 140), & \text{otherwise} \end{cases}$$

Assuming that IBM volatility is 25%, the risk free interest rate is 2.28% p.y. with continuous compounding. Also, assume that IBM will not pay any dividend in that period. Find the price of this contract using risk neutral valuation. **This is not a Binomial Model problem.**

2. Show that the probability that a European call option will be exercised in a risk-neutral world is, with the notation introduced, $N(d_2)$. What is an expression for the value of a derivative that pays off \$100 if the price of a stock at time T is greater than K ?
3. A financial institution has the following portfolio of over-the-counter options on sterling:

| Type | Position | Delta of option | Gamma of option | Vega of option |
|-------------|----------|-----------------|-----------------|----------------|
| <i>Call</i> | 1,000 | 0.50 | 2.2 | 1.8 |
| <i>Call</i> | 500 | 0.80 | 0.6 | 0.2 |
| <i>Put</i> | 2,000 | 0.40 | 1.3 | 0.7 |
| <i>Call</i> | 600 | 0.70 | 1.8 | 1.4 |

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

- (a) What position in the traded option and in sterling would make the portfolio both gamma neutral and delta neutral?

- (b) What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral?
 - (c) Now assume that a second traded option with a delta of 0.1, a gamma of 0.5, and a vega of 0.6 is available. How could the portfolio be made delta, gamma, and vega neutral?
4. A deposit instrument offered by a bank guarantees that investors will receive a return during a 6-month period that is the greater of (a) zero and (b) 40% of the return provided by a market index. An investor is planning to put \$100,000 in the instrument. Describe the payoff as an option on the index. Assuming that the risk-free rate of interest is 8% per annum, the dividend yield on the index is 3% per annum, and the volatility of the index is 25% per annum, is the product a good deal for the investor?
 5. A European call and put option have the same strike price and time to maturity. The call has an implied volatility of 30% and the put has an implied volatility of 25%. What trades would you do?
 6. Consider a portfolio of options on a single asset. Suppose that the delta of the portfolio is 12, the value of the asset is \$10, and the daily volatility of the asset is 2%. Estimate the 1-day 95% VaR for the portfolio from the delta. Suppose next that the gamma of the portfolio is 2.6. Derive a quadratic relationship between the change in the portfolio value and the percentage change in the underlying asset price in one day.
 7. Estimate EWMA and GARCH(1,1) parameters for two exchange rates of your choice.