

1rst Problem Set- Fundamentals
FGV/EBAPE
Asset Pricing

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- 1) Suppose we have three political candidates, A, B, and C, and that there are three voters with preferences as follows (candidates being listed in decreasing order of preference):

- Voter 1: $A B C$
- Voter 2: $B C A$
- Voter 3: $C A B$

Now define societal preferences over $\{A, B, C\}$ as follows: $x \succeq y$ if at least two voters prefer x to y . So, for example, $A \succeq B$ if at least two voters prefer A over B . Using this definition of the weak preference relation, prove (or show that it is false) that these preferences are: (i) complete; (ii) transitive. Be clear and complete in your claims. Can you provide a utility function that represents these preferences?

- 2) In a city center, parking space is rare. Hence, legal parking cost an amount of $t > 0$. Some people decide to park illegally. There is a probability p of being caught which leads to a fine $f > t$. In order to decrease the number of illegal parkers, there are two possible policies: doubling the fine f or doubling the controls, i.e., the probability p . Assuming that implementing both policies have zero cost and that illegal parkers are risk-averse, which is the better policy?
- 3) Daniel Bernoulli and Daniel Kahneman go on vacation. They each have two credit cards and two wallets. With a certain small probability a wallet could be stolen. The probability that a particular wallet be stolen is independent from the probability of another wallet be stolen. Assume that both act according to their theories. Would they put both credit cards into the same wallet or each in a different wallet?
- 4) Consider two assets: a stock and a bond. There are two states of the world (each with probability $1/2$): boom and recession. The stock's return are 8 in a boom and -2% in a recession, the bond yields 2%

each. Compute their mean and variance! Now, find the values of α such that an investor with a mean-variance utility $U(\mu, \sigma) = \mu - \alpha\sigma^2$ is indifferent between both assets! If this investor buys a some stocks (say a proportion $\lambda \in [0, 1]$ of his total invstmen) and some bonds (a proportion $1 - \lambda$). Which λ is optimal for him?

- 5) Assume we have two assets A and B with expected returns of 7% and 10%, and standard deviation of 0.15 and 0.2, for A and B , respectively.
 - If the covariance of A with B is 0.001. Find the expected return and standard deviation of the minimum variance portfolio.
 - If the risk-free interest rate is 0.4%, find the tangent portfolio.
- 6) The IBM stock has expected return 19%, with beta equal to 1.7, on the other hand an GM stock has expected return of 14%, with beta equal to 1.2. Assume that CAPM is valid. What is the expected return of the market portfolio? What is the risk-free interest rate?
- 7) Take any 10 Brazilian stocks.
 - Draw the efficient frontier. Where is located the Ibovespa Index.
 - What you can use as a risk-free interest rate?
 - Compute the stocks' beta in relation to Ibovespa index.