Value at Risk

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Bassel Accords

- Basel I (1988-1996-1999): Market and Credit Risk. Minimal Capital Requirements for Banks
- Basel II (1999-2004-2008): Operational risk and "three pillars" concept – (1) minimum capital requirements, (2) supervisory review and (3) market discipline.

$$\frac{\text{Total capital}}{\text{Credit risk} + \text{Market risk} + \text{Operational risk}} = \text{Bank's capital ratio} > 11\%$$

- Base lll (2010-2013-2018): Bank Liquidity and Leverage: BNP Presentation

The Question Being Asked in VaR

“What loss level is such that we are X% confident it will not be exceeded in N business days?”

VaR and Regulatory Capital

- Regulators base the capital they require banks to keep on VaR
- The market-risk capital is k times the 10-day 99% VaR where k is at least 3.0

VaR and C-VaR

- VaR is the loss level that will not be exceeded with a specified probability
- C-VaR (or expected shortfall) is the expected loss given that the loss is greater than the VaR level
- Although C-VaR is theoretically more appealing, it is not widely used

Advantages of VaR

- It captures an important aspect of risk in a single number
- It is easy to understand
- It asks the simple question: “How bad can things get?”

Time Horizon

- Instead of calculating the 10-day, 99% VaR directly analysts usually calculate a 1-day 99% VaR and assume

$$10\text{-day VaR} = \sqrt{10} \times 1\text{-day VaR}$$

- This is exactly true when portfolio changes on successive days come from independent identically distributed normal distributions

Historical Simulation

- Create a database of the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day
- and so on

The Model-Building Approach

- The main alternative to historical simulation is to make assumptions about the probability distributions of return on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically
- This is known as the model building approach or the variance-covariance approach

Normality Assumption

- If daily returns are normal(μ, σ), then:

VaR_X(Return)=Z(X%) σ- μ

Here μ is the mean daily return and σ is the daily volatility. The constant -Z(X%) is the critical point of the Normal(0,1) for the acumulated area of 1-X%.

Example: X=99, then Z(99%)=2.33 since, N(-2.33)=0.01

Daily Volatilities

- In option pricing we measure volatility "per year"
- In VaR calculations we measure volatility "per day"

$$\sigma_{day} = \frac{\sigma_{year}}{\sqrt{252}}$$

Daily Volatility continued

- Strictly speaking we should define σ_{day} as the standard deviation of the continuously compounded return in one day
- In practice we assume that it is the standard deviation of the percentage change in one day

Microsoft Example

- We have a position worth $10 million in Microsoft shares
- The volatility of Microsoft is 2% per day (about 32% per year)
- We use N=10 and X=99

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Microsoft Example continued

- The standard deviation of the change in the portfolio in 1 day is $200,000
- The standard deviation of the change in 10 days is

$$200{,}000\sqrt{10} = \$632{,}456$$

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Microsoft Example continued

- We assume that the expected change in the value of the portfolio is zero (This is OK for short time periods)
- We assume that the change in the value of the portfolio is normally distributed
- Since $N(-2.33)=0.01$, the VaR is

$$2.33 \times 632{,}456 = \$1{,}473{,}621$$

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AT&T Example

- Consider a position of $5 million in AT&T
- The daily volatility of AT&T is 1% (approx 16% per year)
- The S.D per 10 days is

$$50{,}000\sqrt{10} = \$158{,}144$$

- The VaR is

$$158{,}114 \times 2.33 = \$368{,}405$$

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Portfolio

- Now consider a portfolio consisting of both Microsoft and AT&T
- Suppose that the correlation between the returns is 0.3

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S.D. of Portfolio

- A standard result in statistics states that

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

- In this case σ_X = 200,000 and σ_Y = 50,000 and ρ = 0.3. The standard deviation of the change in the portfolio value in one day is therefore 220,227

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VaR for Portfolio

- The 10-day 99% VaR for the portfolio is
  $$220{,}227 \times \sqrt{10} \times 2.33 = \$1{,}622{,}657$$
- The benefits of diversification are (1,473,621+368,405)–1,622,657=$219,369

Stress Testing

This involves testing how well a portfolio performs under some of the most extreme market moves seen in the last 10 to 20 years

Back-Testing

- Tests how well VaR estimates would have performed in the past
- We could ask the question: How often was the actual 10-day loss greater than the 99%/10 day VaR?